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Early acceleration of students in mathematics: Does it promote growth and stability of growth in achievement across mathematical areas?☆

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Abstract

Using data from the Longitudinal Study of American Youth (LSAY), the present study examined whether early acceleration of students into formal algebra at the beginning of middle school promoted evident growth in different mathematical areas (basic skills, algebra, geometry, and quantitative literacy) and stable growth across these mathematical areas. Results of multivariate multilevel analyses showed that low achieving students who were accelerated into formal algebra grew faster than not only low achieving students who were not accelerated but also high achieving students who were not accelerated. The rates of growth of accelerated low achieving students were even comparable to those of accelerated high achieving students. All low achieving students showed the same potential to take advantage of early acceleration regardless of their individual, family, and school characteristics. Early acceleration also promoted stability of growth across mathematical areas, and this stability was not dependent on student and school characteristics.

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1. Introduction

The present study examined whether early acceleration of students in mathematics results in evident growth in various mathematical areas (basic skills, algebra, geometry, and quantitative literacy) and balanced growth across these mathematical areas, using data from the Longitudinal Study of American Youth (LSAY), a six-year (Grades 7–12) panel study of mathematics (and science) education of public middle and high school students (Miller, Kimmel, Hoffer, & Nelson, 2000). In the literature, early acceleration is usually defined as the educational practice to instruct students with advanced learning materials that are reserved for students at higher grade levels. In the present study, early acceleration is specifically defined as early access to formal algebra (Algebra I) at the beginning of middle school (Grades 7 and 8).

The present study attempted to determine how much growth in achievement accelerated students can demonstrate in each of the four mathematical areas in relation to their initial mathematics achievement at the beginning of middle school and examine whether accelerated students who have a faster rate of growth in achievement in one mathematical area also have a faster rate of growth in achievement in other mathematical areas. Furthermore, the present study attempted to examine the impacts of student and school characteristics on growth in each of the four mathematical areas and stability of growth across the four mathematical areas among accelerated students compared with non-accelerated students.

Such a study is important in that researchers who study mathematics coursework have increasingly realized the importance of timing in taking mathematics courses. Spring (1989) describes schools as a "great sorting machine" in which selecting one mathematics course rather than another at a particular time during a student's academic career may function as a critical gateway in the successful learning of mathematics. Formal algebra is a required course for all advanced mathematics and sciences courses, and as such it is the first major gateway that determines student subsequent high school mathematics and science experiences (Usiskin, 1995; Wagner & Kieran, 1989). Smith (1996) concluded that "early access to algebra has an effect beyond simple increased knowledge measures and, in fact, may 'socialize' a student into taking more mathematics, regulating access both to advanced coursework and increased achievement in high school" (p. 141).

Why is formal algebra so important to the learning of mathematics? Kieran (1992) proposed a historical-epistemological framework to understand the role of formal algebra in mathematics. Mathematics has long been characterized as abstract, representing a move "from ordinary language descriptions of problem situations and their solutions to symbolic representations and procedures" (Kieran, 1992, p. 390), and she believes that formal algebra most appropriately highlights the gradual loss in the meaning of words during such a move. Algebraic symbolism is then the essential building block for mathematics. Boero, Douek, and Ferrari (2002) as well as Kleiner (1989) employed such examples as mathematical computing, mathematical function, and analytical geometry to demonstrate that the creation of a symbolic (formal) algebra is essential to the development of other mathematical concepts and procedures.

Despite the recognition of the importance of formal algebra, when the most appropriate time is to encourage or even push students to consider taking formal algebra is a quite controversial policy and practice issue. Some researchers believe that it is beneficial and important for students to take formal algebra as early as Grade 8 (e.g., Usiskin, 1987). The main reason given by this group of researchers for early access to formal algebra is that it opens doors to all students for increased learning opportunities and attainments in mathematics (see Oakes, 1990; Smith, 1996; Useem, 1993). Mills, Ablard, and Gustin (1994) argued that traditionally higher-level curriculum can be instructed at a much earlier age because many students reach the Piagetian "formal operational stage" much sooner than normally expected.

Other researchers believe that it is ineffective and even harmful for students to take formal algebra at the beginning of middle school (e.g., Frevost, 1985). The main reason given by this group of researchers against early access to formal algebra is that it frustrates and alienates students, even mathematically talented students, for further studies in mathematics (see Karper & Melnick, 1993; Liu & Liu, 1997). Advocating curricular enrichment as an alternative, these researchers fear that early acceleration of students into formal algebra leads to array of cognitive and affective difficulties among students and burns them out eventually.

To provide empirical evidence to facilitate this debate, the present study compared students who were accelerated and not accelerated into formal algebra, with a focus on rates of growth in the four mathematical areas and stability of growth across the four mathematical areas between these two groups of students. The analytical logic of the present study is as follows.

(a) If students who are accelerated into formal algebra at the beginning of middle school grow faster (slower) (across middle and high school grades) in achievement in a mathematical area than students who are not accelerated, then early acceleration into formal algebra promotes (hampers) growth in the mathematical area.
(b) If students who are accelerated into formal algebra at the beginning of middle school demonstrate a greater (lesser) degree of stability in growth (across middle and high school grades) among different mathematical areas than students who are not accelerated, then early acceleration into formal algebra promotes (hampers) stability of growth across different mathematical areas.

This analytical logic guided the examination of the following research questions in the present study:

1. How much growth does early acceleration of students into formal algebra promote in each of the four mathematical areas?
2. Do student and school characteristics moderate the effects of early acceleration into formal algebra in each of the four mathematical areas?
3. Does early acceleration into formal algebra promote the stability of growth in achievement across the four mathematical areas?
4. Do student and school characteristics moderate the stability of growth in achievement across the four mathematical areas?

The present study examined these issues of early acceleration of students in mathematics in relationship to their initial mathematics achievement at the beginning of middle school. This effort underscores the assumption that early acceleration into formal algebra may have different impacts on students of differing academic background.

Results of the present study therefore are able to provide useful empirical evidence as to the effectiveness (or ineffectiveness) of early acceleration into formal algebra from a longitudinal perspective (across the entire middle and high school). In this sense, the present study responds to the major limitation of research in early acceleration of students in mathematics as Kolitch and Brody (1992) have noted that "studies are lacking that would provide a comprehensive picture of the mathematics preparation of [accelerated] talented students throughout the secondary school years" (p. 79). While this comprehensive picture is unlikely to be painted in a single study, the present study is able to show whether early acceleration of students into formal algebra promotes significant academic growth across different mathematical areas and creates balanced academic development across different mathematical areas throughout the entire secondary school years. Findings of the present study are able to assist politicians, administrators, and researchers to assess the merits and deficiencies of the policy and practice of early acceleration.

2. Method

2.1. Data

The Longitudinal Study of American Youth (LSAY) contained a national probability sample of 51 pairs of middle and high schools drawn through a stratified sampling framework from a national population of middle and high schools in 12 sampling strata defined on the basis of geographic region and type of community (Miller et al., 2000). About 60 seventh graders were then randomly selected from each sampled school and these seventh graders were studied for six years (from Grades 7 to 12). The sample contained 3116 students in the 7th grade, 2798 in the 8th grade, 2748 in the 9th grade, 2583 in the 10th grade, 2409 in the 11th grade, and 2215 in the 12th grade. Student emigration and dropout were the major reason for data attrition. Students took a mathematics (and science) achievement test and completed a student questionnaire each year during the six-year period of study.

2.2. Measures

The LSAY data are particularly well suited to the purposes of the present study because it is the most comprehensive longitudinal database among all existing national education databases, with coverage over the entire secondary school grades from 7 to 12 and a focus on mathematics (and science) education. Outcome measures (dependent variables) were mathematics achievement in four mathematical areas or subscales as mentioned earlier (basic skills, algebra, geometry, and quantitative literacy). The basic skills subscale measured students' ability to solve problems associated with number concepts and operations. The algebra subscale measured students' ability to solve problems typical of a first algebra course in high school. The geometry subscale measured students' ability to solve problems in geometry and measurement. The quantitative literacy subscale measured students' ability to solve problems associated with percentage, probability, data analysis, and graph reading.

Each subscale was composed of items directly adopted from the National Assessment of Educational Progress (NAEP). The same subscale was used across the period of six years (Grades 7 to 12). Scores were calibrated (and thus became comparable) across this period, using item response theory (IRT) (Miller et al., 2000). Specifically, the LSAY staff used a three-parameter IRT model to adjust scores for item difficulty, reliability, and guessing. IRT scores have several advantages over (raw) total scores (see, for example, Crocker & Algina, 1986). One advantage particularly relevant to the present study is that IRT scores represent a refined interval scale of measurement very important to examine growth or change over time. Scores in Grade 7 were scaled to a metric with a mean of 50 and a standard deviation of 10, and students were scored on the same metric in subsequent years (Miller et al., 2000). Based on this procedure, a measure of growth in each of the four mathematical subscales could be estimated, using a growth model approach.

Status of early acceleration in mathematics and initial mathematics achievement at the beginning of middle school were the key independent variables. Based on detailed coursework information on each student in the LSAY, one variable was created to denote status of early acceleration of students in mathematics. Students who took Algebra I in either Grade 7 or Grade 8 were defined as accelerated in mathematics. These students had early access to formal algebra. Meanwhile, students who took Algebra I in neither Grade 7 nor Grade 8 were defined as not accelerated in mathematics. These students did not have early access to formal algebra.

Besides the four separate subscales in the mathematics test (discussed above), the LSAY staff also combined them into an overall measure of mathematics achievement in each year. Similarly, scores were scaled to a metric with a mean of 50 and a standard deviation of 10, with IRT procedures. Mathematics achievement in Grade 7 was used in the present study to represent students' initial mathematics achievement at the beginning of middle school. Cronbach's α for the whole mathematics achievement test was 0.86 in Grade 7.

Other independent variables came from the student questionnaire used in the LSAY and described student characteristics, including gender, age, parent socioeconomic status (SES), race-ethnicity, family structure (number of parents), family size (number of siblings), and language spoken at home. Gender was coded into one dichotomous variable, with male students as the base line effect against which female students were compared. Age was coded as a continuous variable, with one month as the measurement unit. Parent SES was a combined index of parent-reported education and occupation as well as student-reported household possessions and was scaled into two standardized continuous variables denoting father's SES and mother's SES.

Race-ethnicity was coded into four dichotomous variables, with White students as the base line effect against which Hispanic, Black, Asian, and Native American students were compared. Family structure was coded into one dichotomous variable, with students from both-parent households as the base line effect against which students from single-parent households were compared. Family size was coded as a continuous variable, with one person as the measurement unit. Language spoken at home was coded into one dichotomous variable, with students speaking English at home as the base line effect against which students speaking other languages at home were compared. For the purpose of statistical analysis, these variables descriptive of student characteristics were either centered (in the cases of all dichotomous variables as well as age and family size) or standardized (in the cases of father's SES and mother's SES).

School-level variables came from the student, teacher, and principal questionnaires used in the LSAY and described school contextual and climatic characteristics. School-level variables were selected in the present study following the theoretical scheme on school effectiveness. Teddlie and Reynolds (2000) have described how schools influence learning outcomes of students through differing school context and climate. Some school-level variables were aggregated measures to the school level from student and teacher questionnaires.

School contextual variables included school size, school location (two dummy variables denoting suburban and rural with urban as the base-line effect), school socioeconomic composition (measured through the percentage of free-lunch students), school racial-ethnic composition (measured through the percentage of minority students), grade span, student–teacher ratio, teacher education level, and teacher experience in mathematics.

School climatic variables (with Cronbach's α reported in parentheses in case of composite variables) included computer-student ratio in mathematics, academic expectation (0.69), disciplinary climate (0.77), parental involvement, principal leadership (0.86), teacher autonomy (0.79), teacher commitment (0.58), staff cooperation (0.51), mathematics homework, general support for mathematics (0.77), and extracurricular activities (0.70). The use of partial scales of these composite variables was explored as a way to improve reliability. Eventually, full scales were maintained because partial scales showed trivial or no improvement in reliability. Note that "low reliability does not necessarily mean lack of precision" in measurement (Rogosa, Brandt, & Zimowski, 1982, p. 744). It is possible to encounter low reliability when school-level responses are not reasonably distinguishable among schools (similar situations are not uncommon when measuring social phenomena). The appendix contains descriptions of these school climatic variables. For the purpose of statistical analysis, all school-level variables were centered.

2.3. Stability measures

Because a measure of growth (from Grades 7 to 12) could be estimated in each of the four mathematical subscales, when these measures of growth across the four mathematical subscales were kept together in a single statistical equation (a multivariate framework), correlations of these measures of growth could be estimated as measures of stability of growth across the four mathematical subscales (which were used to compare against the analytical logic of research as discussed earlier).

A correlation as a measure of stability simply indicates that growth is in the same direction across two mathematical subscales. For example, a perfect positive correlation happens when the rank of growth for each student is exactly the same between two subscales, whereas a perfect negative correlation happens when the rank of growth for each student is totally reversed between two subscales. In this sense, stability in the present study did not necessarily mean the exact amount of growth but rather the exact rank of growth.

Naturally, students are not expected to grow at the same rate across mathematical areas due to different levels of abstraction and difficulty associated with different mathematical areas. However, the present study is indeed in a position to examine whether early acceleration of students in mathematics can produce a similar amount of addition (either positive or negative) to growth across the four mathematical areas. Such information can be captured in standardized regression coefficients associated with status of early acceleration, initial mathematics achievement, and their interaction (assuming that mathematics achievement at the beginning of middle school is critically important to early acceleration). Therefore, the present study adopted two measures of stability: correlation coefficients that measure the consistency in position in growth and (standardized) regression coefficients that measure the amount of addition to growth due to early acceleration of students in mathematics.

2.4. Statistical analysis

The kind of analysis as discussed above could be performed within the framework of hierarchical linear modeling (HLM) (Raudenbush & Bryk, 2002). Similar modeling ideas have been presented in, for example, Raudenbush, Rowan, and Kang (1991). Therefore, HLM models were employed to quantify the amount of growth (from Grades 7 to 12) in each of the four mathematical areas and examine the degree of stability of growth in achievement across the four mathematical areas. Specifically, these models investigated whether early acceleration of students into formal algebra promoted growth and stability of growth across different mathematical areas.

A four-level HLM model was developed that represents a multivariate, multilevel analytic approach. The level-one model integrates scores from the four mathematical areas into a single equation with four dichotomous variables denoting the four mathematical subscales:

$$Y_{ijkl} = \psi_{1jkl}X_{1ijkl} + \psi_{2jkl}X_{2ijkl} + \psi_{3jkl}X_{3ijkl} + \psi_{4jkl}X_{4ijkl},$$

$$X_{sijkl} = \begin{cases} 1 & s = i, \\ 0 & s \neq i, \end{cases}$$

where Y_{ijkl} is the subscale i score in grade level j for student k in school l. This model without intercept and error term is a measurement device to combine scores from the four subscales into one equation so that variance and covariance components in growth among the four subscales can be computed at the student and school levels. Note that covariance components in growth cannot be obtained with the univariate approach of analysis (four separate HLM models). As a measurement device, this model generates no meaningful statistics but constitutes one level to create a multivariate environment in which hierarchical (3-level) data with repeated measures nested within students nested within schools can be analyzed. With no error term in the model, Y_{ijkl} is not the predicted value as normally assumed in regression but the actual score (subscale i score in grade level j for student k in school l).

The level-two model estimates rates of growth across the four mathematical subscales. It includes four sets of linear regression equations that model students' scores on their grade levels in each subscale.

$$\psi_{1jkl} = \beta_{10kl} + \beta_{11kl}T_{jkl} + \varepsilon_{1jkl},$$
$$\psi_{2jkl} = \beta_{20kl} + \beta_{21kl}T_{jkl} + \varepsilon_{2jkl},$$
$$\psi_{3jkl} = \beta_{30kl} + \beta_{31kl}T_{jkl} + \varepsilon_{3jkl},$$
$$\psi_{4jkl} = \beta_{40kl} + \beta_{41kl}T_{jkl} + \varepsilon_{4jkl},$$

where ψ_{1jkl} to ψ_{4jkl} are subscale scores (basic skills, algebra, geometry, and quantitative literacy) in grade level j for student k in school l. T_{jkl} is the grade level (j) in which student k in school l is, and ε_{1jkl} to ε_{4jkl} are error terms. Parameters β_{11kl} to β_{41kl} represent rates of growth in the four subscales for student k in school l.

The level-three model examines the effects of student characteristics on rates of growth in the four mathematical subscales. It contains four sets of regression equations modeling the rates of growth, respectively, with student characteristics.

$$\beta_{11kl} = \gamma_{110l} + \sum \gamma_{11pl}X_{pkl} + u_{11kl},$$
$$\beta_{21kl} = \gamma_{210l} + \sum \gamma_{21pl}X_{pkl} + u_{21kl},$$
$$\beta_{31kl} = \gamma_{310l} + \sum \gamma_{31pl}X_{pkl} + u_{31kl},$$
$$\beta_{41kl} = \gamma_{410l} + \sum \gamma_{41pl}X_{pkl} + u_{41kl}.$$

In these equations, students' rates of growth are represented as school average rates of growth (γ_{110l} to γ_{410l}), error terms unique across students (u_{11kl} to u_{41kl}), and contributions (more precisely, adjustment, in the present case) of student characteristics. This level-three model allows rates of growth and stability of growth to be estimated unconditionally (without student characteristics) and conditionally (with adjustment for student characteristics).

Finally, the level-four model examines the effects of school characteristics on school average rates of growth in the four mathematical subscales. Statistically, it contains four regression equations modeling school average rates of growth, respectively, with school characteristics.

$$\gamma_{110l} = \varpi_{1100} + \sum \varpi_{11pl} W_{pl} + v_{110l},$$
$$\gamma_{210l} = \varpi_{2100} + \sum \varpi_{21pl} W_{pl} + v_{210l},$$
$$\gamma_{310l} = \varpi_{3100} + \sum \varpi_{31pl} W_{pl} + v_{310l},$$
$$\gamma_{410l} = \varpi_{4100} + \sum \varpi_{41pl} W_{pl} + v_{410l}.$$

In these equations, school average rates of growth are represented as grand average rates of growth (ϖ_{1100} to ϖ_{4100}), error terms unique across schools (v_{110l} to v_{410l}), and contributions (again, more precisely in the present case, adjustment) of school characteristics. This level-four model allows school average rates of growth and stability of average growth to be estimated unconditionally (without school characteristics) and conditionally (with adjustment for school characteristics).

All HLM analyses were performed on the PC platform of the MLwiN program (Rasbash et al., 2000). To prepare data for HLM analyses, four data files were originally created on the PC platform of SPSS (one for each level) and then merged into a single data file (see Rasbash et al., 2000). The MLwiN program can directly read this merged SPSS data file for HLM analyses.

3. Results

Rates of growth were estimated for the four mathematical areas (basic skills, algebra, geometry, and quantitative literacy). These rates of growth were computed in four different ways. First, unconditional rates of growth were estimated without any adjustment. Conditional rates of growth were then estimated with early acceleration (together with initial mathematics achievement and their interaction), student characteristics, and school characteristics sequentially and accumulatively added to the unconditional model. A comparison between unconditional and conditional rates of growth provided a measure of the impacts of early acceleration, student characteristics, and school characteristics on rates of growth in the four mathematical areas. The same logic was also used for estimating stability of growth across the four mathematical areas.

Table 1 presents unconditional annual rates of growth in and correlations of growth among the four mathematical areas. On average, students grew annually at about 4 points in basic skills, 8 in algebra, 6 in geometry, and 4 in quantitative literacy. These rates of growth were all statistically significant. It is obvious from these standardized coefficients (as measures of stability) that students grew somewhat faster in algebra and geometry than in basic skills and quantitative literacy. However, correlations of growth were positive and substantial at both student and school levels, indicating that the ranks of growth among students and schools were fairly consistent across the four mathematical areas. In other words, students (schools) who grew faster in basic skills also grew faster in algebra, geometry, and quantitative literacy. The highest correlation was between algebra and geometry (0.96) among students and between algebra and quantitative literacy (0.98) among

Table 1
Annual rates of growth in and correlation coefficients of growth among basic skills, algebra, geometry, and quantitative literacy

	Growth	*SE*	1	2	3	4
1. Basic skills	3.73	0.11		0.84	0.84	0.90
2. Algebra	7.73	0.22	0.91		0.97	0.98
3. Geometry	6.21	0.21	0.90	0.96		0.94
4. Quantitative literacy	3.82	0.12	0.88	0.91	0.90	

Note. All rates of growth are statistically significant at the α level of 0.05. Correlations at the student level are presented in the lower triangle. Correlations at the school level are presented in the upper triangle.

schools. The lowest correlation (still considerably strong) was between basic skills and quantitative literacy (0.88) among students and between basic skills and both algebra (0.84) and geometry (0.84) among schools.

3.1. Does early acceleration promote growth and stability of growth?

Table 2 presents the effects of early acceleration of students in mathematics on annual rates of growth in and correlations of growth among the four mathematical areas. Because mathematics achievement at the beginning of middle school during which period early acceleration occurs may be critical, initial (Grade 7) mathematics achievement and its interaction with early acceleration were also modeled.

The interaction between early acceleration and initial mathematics achievement was statistically significant across all mathematical areas (note that main effects cannot be interpreted when interaction effects are statistically significant). These significant, negative interaction effects indicated that the difference in the rate of growth between accelerated students and non-accelerated students was significantly greater among students with low initial mathematics achievement than students with high initial mathematics achievement across all mathematical areas, particularly in algebra. In other words, success of early acceleration of students into formal algebra (as measured through growth in the four mathematical areas during the entire mid-

Table 2
Effects of early acceleration of students in mathematics on annual rates of growth in and correlation coefficients of growth among basic skills, algebra, geometry, and quantitative literacy

	A. Acceleration		B. Achievement		A × B		Correlation			
	Effect	*SE*	Effect	*SE*	Effect	*SE*	1	2	3	4
1. Basic skills	2.02	0.23	1.67	0.06	−1.26	0.18		0.66	0.65	0.90
2. Algebra	4.28	0.37	1.77	0.10	−2.22	0.29	0.76		0.95	0.90
3. Geometry	3.12	0.32	1.85	0.08	−1.03	0.26	0.80	0.93		0.83
4. Quantitative literacy	1.83	0.23	1.44	0.06	−0.97	0.19	0.90	0.83	0.86	

Note. All effects are statistically significant at the α level of 0.05. Correlations at the student level are presented in the lower triangle. Correlations at the school level are presented in the upper triangle.

dle and high school) depended on mathematics achievement at the beginning of middle school.

To examine this issue more closely, annual rates of growth were calculated in the four mathematical areas (see Table 3), based on regression coefficients of early acceleration, initial mathematics achievement, and their interaction as reported in Table 2. Overall, students who were accelerated into formal algebra at the beginning of middle school (Grades 7 and 8) grew faster than students who were not accelerated in all mathematical areas regardless of their mathematics achievement at the beginning of middle school. Consider the example of algebra. Among students with high initial mathematics achievement, accelerated ones grew at an annual rate of 12 points, whereas non-accelerated ones grew at an annual rate of 10 points. Among students with low initial mathematics achievement, accelerated ones grew at an annual rate of 13 points, whereas non-accelerated ones grew at an annual rate of 8 points.

However, it is clear that students with low initial mathematics achievement benefited more from early acceleration than students with high initial mathematics achievement. Specifically, early acceleration was related to an advantage of 3 points in annual growth in geometry regardless of initial mathematics achievement. In both basic skills and quantitative literacy, early acceleration was related to no advantage in annual growth among high initial achievers but an advantage of 2 points among low initial achievers. Early acceleration was related to an advantage of 2 points in annual growth in algebra among high initial achievers but an advantage of 5 points among low initial achievers. Furthermore, annual growth rates of accelerated low initial achievers certainly surpassed those of non-accelerated high initial achievers and paled even those of accelerated high initial achievers.

Table 3
Annual rates of growth in basic skills, algebra, geometry, and quantitative literacy conditional on early acceleration and mathematics achievement

A. Early acceleration	B. Mathematics achievement (at the beginning of middle school)	
	High ($B = 1$)	Low ($B = 0$)
Basic skills		
Accelerated ($A = 1$)	6.55	6.14
Not accelerated ($A = 0$)	5.79	4.12
Algebra		
Accelerated ($A = 1$)	12.14	12.95
Not accelerated ($A = 0$)	10.08	8.31
Geometry		
Accelerated ($A = 1$)	10.55	9.73
Not accelerated ($A = 0$)	8.46	6.61
Quantitative literacy		
Accelerated ($A = 1$)	6.44	5.97
Not accelerated ($A = 0$)	5.58	4.14

Note. One standard deviation is used to represent mathematics achievement at the beginning of middle school (Grade 7) with 1 denoting high achievement and 0 denoting low achievement.

Comparing Tables 1 and 2, the range in correlation coefficients among students was from 0.88 to 0.96 before early acceleration (Table 1) was adjusted and from 0.76 to 0.93 after early acceleration was adjusted (Table 2). The range in correlation coefficients among schools was from 0.84 to 0.98 before early acceleration was adjusted and from 0.65 to 0.95 after early acceleration was adjusted. The impact of early acceleration was greater on the stability of growth among schools than among students, indicating that early acceleration was related to more changes in rank in growth among schools than among students. Among both students and schools, the relationship of basic skills to algebra and geometry was most influenced by early acceleration. Specifically, early acceleration reduced the stability of growth between basic skills and algebra as well as between basic skills and geometry.

Overall, changes in correlation between Tables 1 and 2 were fairly marginal. In other words, early acceleration of students in mathematics only slightly altered the stability of growth across the four mathematical areas with correlations remaining strong, in particular among students. If accelerated students grew significantly faster than non-accelerated students while correlations changed in a marginal way between Tables 1 and 2, early acceleration ought to result in consistent upgrades in rank in rates of growth for accelerated students across the four mathematical areas.

Finally, although early acceleration was related to consistent changes in rank in rates of growth, standardized coefficients associated with status of early acceleration (reported as effects) in Table 2 were different across the four mathematical areas, indicating that the amount of addition to growth resulting from early acceleration of students into formal algebra at the beginning of middle school was different. Obviously in Table 2, early acceleration added somewhat more growth to algebra and geometry than to basic skills and quantitative literacy.

3.2. Do student and school characteristics affect growth and stability of growth?

Table 4 presents the effects of early acceleration of students into formal algebra on the annual rates of growth in the four mathematical areas and correlation coefficients

Table 4
Effects of early acceleration of students in mathematics on annual rates of growth in and correlation coefficients of growth among basic skills, algebra, geometry, and quantitative literacy, conditional on student characteristics

	A. Acceleration		B. Achievement		A × B		Correlation			
	Effect	*SE*	Effect	*SE*	Effect	*SE*	1	2	3	4
1. Basic skills	1.96	0.22	1.59	0.06	−1.22	0.18		0.70	0.68	0.91
2. Algebra	4.16	0.36	1.64	0.10	−2.14	0.29	0.76		0.94	0.91
3. Geometry	3.02	0.32	1.79	0.09	−0.98	0.25	0.79	0.92		0.84
4. Quantitative literacy	1.76	0.23	1.40	0.06	−0.95	0.18	0.90	0.82	0.85	

Note. All effects are statistically significant at the α level of 0.05. Correlations at the student level are presented in the lower triangle. Correlations at the school level are presented in the upper triangle. Correlations are adjusted for student characteristics (gender, age, father socioeconomic status (SES), mother SES, race-ethnicity, family structure, family size, and language spoken at home).

of the rates of growth among the four mathematical areas conditional on student characteristics. A comparison between Tables 2 and 4 shows that student characteristics did not modify the stability of growth across the four mathematical areas (as established by early acceleration) in any significant way. Some marginal changes did occur in correlation after student characteristics were adjusted. The largest change in correlation was 0.01 among students and 0.04 among schools. These changes were too trivial to be taken seriously for policy or practice purposes. Overall, the presence of student characteristics made little difference over and above early acceleration in terms of the stability of growth across the four mathematical areas among both students and schools.

Similarly, student characteristics did not modify in any significant way the effects of early acceleration on the annual rates of growth in any mathematical areas. Interaction effects and main effects associated with early acceleration and initial mathematics achievement were fairly similar between Tables 2 and 4. Such analytical results clearly indicated that student characteristics did not influence much the effects of early acceleration on the annual rates of growth in any mathematical areas (including the amount of addition to growth resulting from early acceleration used as another measure of stability of growth across mathematical areas).

Table 5 presents the effects of early acceleration of students into formal algebra on the annual rates of growth in the four mathematical areas and correlation coefficients of these rates of growth among the four mathematical areas conditional on both student and school characteristics. Correlations were almost identical at the student level and fairly similar at the school level between Tables 4 and 5. There was an isolated case with a difference in correlation of 0.10 between basic skills and geometry. The change in correlation was quite marginal even in this case. Overall, school

Table 5
Effects of early acceleration of students in mathematics on annual rates of growth in and correlation coefficients of growth among basic skills, algebra, geometry, and quantitative literacy, conditional on student and school characteristics

	A. Acceleration		B. Achievement		A × B		Correlation			
	Effect	*SE*	Effect	*SE*	Effect	*SE*	1	2	3	4
1. Basic skills	1.99	0.22	1.58	0.06	−1.21	0.18		0.67	0.78	0.93
2. Algebra	4.19	0.36	1.64	0.10	−2.11	0.29	0.76		0.95	0.91
3. Geometry	3.02	0.32	1.77	0.08	−0.97	0.25	0.79	0.92		0.88
4. Quantitative literacy	1.78	0.23	1.39	0.06	−0.94	0.18	0.89	0.82	0.85	

Note. All effects are statistically significant at the α level of 0.05. Correlations at the student level are presented in the lower triangle. Correlations at the school level are presented in the upper triangle. Correlations are adjusted for student characteristics (gender, age, father socioeconomic status (SES), mother SES, race-ethnicity, family structure, family size, and language spoken at home) as well as school contextual characteristics (school size, school location, school socioeconomic composition, school racial-ethnic composition, grade span, student–teacher ratio, teacher education level, and teacher experience in mathematics) and school climatic characteristics (computer-student ratio in mathematics, academic pressure or expectation, disciplinary climate, parental involvement, principal leadership, teacher autonomy, teacher commitment, staff cooperation, mathematics homework, general support for mathematics, and extracurricular activities).

characteristics did not modify over and above student characteristics the stability of growth across the four mathematical areas among either students or schools.

As easily seen, the effects associated with early acceleration, initial mathematics achievement, and their interaction were extremely similar between Tables 4 and 5 across all mathematical areas. Clearly, school characteristics did not influence much over and above student characteristics the effects of early acceleration on the annual rates of growth in any mathematical areas.

A comparison between Tables 2 and 5 shows that student and school characteristics together did not modify the stability of growth across the four mathematical areas (as established by early acceleration) in any significant way either. There were some marginal changes in correlation after student and school characteristics were adjusted. The largest change in correlation was 0.01 among students and 0.05 among schools (excluding the isolated case of 0.13 between basic skills and geometry at the school level). These changes in correlation (even after counting in 0.13) were hardly meaningful from the perspective of education policies or practices. In general, the presence of both student and school characteristics did not make much difference in the stability of growth across the four mathematical areas (as established by early acceleration) among either students or schools.

Neither did student and school characteristics together modify in any significant way the effects of early acceleration of students into formal algebra on the annual rates of growth in the four mathematical areas. Interaction effects as well as main effects associated with early acceleration and initial mathematics achievement were fairly similar between Tables 2 and 5. Therefore, student and school characteristics together did not influence much the effects of early acceleration on the annual rates of growth in any mathematical areas (including the amount of addition to growth resulting from early acceleration used as another measure of stability of growth across the four mathematical areas).

4. Discussion

4.1. How much growth does early acceleration into formal algebra promote?

The analytical logic of research in the present study defines the effectiveness of early acceleration of students into formal algebra at the beginning of middle school according to a comparison on rates of growth in four mathematical areas (basic skills, algebra, geometry, and quantitative literacy) during the entire middle and high school between students who were accelerated and not accelerated. Specifically, the rates of growth in the four mathematical areas (Table 2) were used as the baseline data of early acceleration against which the impact of student and school characteristics on early acceleration was studied.

Following such a logic, it was found that students with different initial mathematics achievement benefited differently from early acceleration. Low achieving students who were accelerated into formal algebra at the beginning of middle school grew not only faster than low achieving students who were not accelerated into formal algebra but also faster than high achieving students who were not accelerated into formal algebra. As a matter of fact, the rates of growth of accelerated low achieving students were even comparable to those of accelerated high achieving students.

Early acceleration of low achieving students into formal algebra at the beginning of middle school appears to be a more successful case than early acceleration of high achieving students. The advantage of early acceleration was the same between low and high achieving students in geometry but greater among low than high achieving students in basic skills, quantitative literacy, and particularly algebra. Recall that the advantage of early acceleration was 3 points annually for both low and high achieving students in geometry, 2 points annually versus zero between low and high achieving students in both basic skills and quantitative literacy, and 5 versus 2 points annually between low and high achieving students in algebra.

There might be a "ceiling" phenomenon in basic skills and quantitative literacy for high achieving students. Such basic mathematical topics as number concept, number operation, percentage, probability, data analysis, and graph reading are very likely to have been mastered well no matter high achieving students are accelerated or not into formal algebra at the beginning of middle school. What this argument implies is that the lack of substantial growth in basic skills and quantitative literacy may not be used as indicators of failure for early acceleration of high achieving students into formal algebra at the beginning of middle school.

Relatively, algebra and geometry represent more challenging mathematical topics and should be the focus when assessing the effectiveness of early acceleration of students into formal algebra at the beginning of middle school. Following this logic, early acceleration of low achieving students is particularly justified given that accelerated low achieving students demonstrated an equivalent rate of growth to accelerated high achieving students in geometry (3 points annually) and even a faster rate of growth than accelerated high achieving students in algebra (5 versus 2 points annually).

It appears that early acceleration of low achieving students into formal algebra at the beginning of middle school may be an effective strategy to improve mathematics achievement of students in this academic category. The superior growth of accelerated low achieving students across all mathematical areas to that of non-accelerated low achieving students is very tempting to make this suggestion. Unlike high achieving students who earn few merits by growing well in basic skills and quantitative literacy (see discussion earlier), it is a great success in itself for low achieving students to progress so well in basic skills and quantitative literacy. This means solid foundations for low achieving students to pursue more advanced mathematics studies. It is certainly an extra success (perhaps unexpected to many researchers) that low achieving students could also grow so well in algebra and geometry once they were accelerated into formal algebra at the beginning of middle school.

One concern about accelerating low achieving students is that the fast-pace of learning may compromise a solid mastery of such basic mathematical topics as number concept, number operation, percentage, probability, data analysis, and graph reading. This concern appears to be unnecessary in that accelerated low achieving students demonstrated healthy rates of growth in both basic skills and quantitative literacy. Overall, early access to formal algebra did not appear to burn accelerated low achieving students out. Instead, early acceleration of low achieving students into formal algebra at the beginning of middle school seems to be fairly effective in promoting growth across mathematical areas.

Different hypotheses can be offered to explain why early acceleration of students into formal algebra is important to growth in mathematics achievement, especially for low achieving students. One hypothesis can be directly framed based on the work of Alexander (1997, 2003). The challenge of more advanced mathematical problems that students regularly encounter in early access to formal algebra prevents boredom and motivates them to invest more effort into the learning of mathematics. In particular, students' individual interest in mathematics is likely to rise relative to problems that are matched well to their level of domain knowledge in mathematics (Alexander, 1997, 2003). As a result, "early access to algebra [may regulate] access both to advanced coursework and increased achievement in high school" (Smith, 1996, p. 141). Analytical results of the present study demonstrate that such a challenge, if institutionalized at the beginning of middle school, can benefit particularly low achieving students.

4.2. Do student and school characteristics moderate effects of early acceleration?

Overall, the conclusion from comparing Tables 4 and 5 with Table 2 is that student and school characteristics did not make much difference in the rates of growth in any of the four mathematical areas. With fairly similar interaction effects and main effects across Tables 2, 4, and 5, advantages of early acceleration that low achieving students demonstrated were hardly influenced by student and school characteristics. This conclusion is a particularly important endorsement for accelerating low achieving students because analytical results indicate that the advantages in the rates of growth of low achieving students who were accelerated into formal algebra at the beginning of middle school were not related to individual differences and school effects. Stated differently, once low achieving students were accelerated into formal algebra at the beginning of middle school, they tended to grow equally well regardless of their individual, family, and school characteristics. This evidence suggests that all low achieving students have the same potential to take advantage of early acceleration into formal algebra at the beginning of middle school.

4.3. Does early acceleration into formal algebra promote stability of growth?

The present study used correlations among rates of growth in four mathematical areas as a measure of stability (or consistency) of growth across mathematical areas. The analytical logic of research is that unconditional correlations of rates of growth among students and schools (Table 1) were used as the base line data against which conditional correlations adjusted for the effects of early acceleration of students into formal algebra at the beginning of middle school were compared. Table 1 clearly indicates that there was considerable stability of growth across the four mathematical areas among both students and schools, with most correlations above 0.90.

Students who grew faster in one mathematical area also grew faster in other mathematical areas. Recall that correlation as a measure of stability indicates the consistency in rank rather than amount of growth. Therefore, students' ranks or positions in rates of growth were fairly consistent across the four mathematical areas.

After early acceleration of students into formal algebra at the beginning of middle school was considered, correlations in Table 2 were similar to those in Table 1. The largest changes in correlation between Tables 2 and 1 are 0.15 among students and 0.19 among schools. Most changes in correlation are well below 0.10. Obviously, even the largest correlations of 0.15 (at the student level) and 0.19 (at the school level) fall somewhat short of being practically meaningful. Early acceleration of students into formal algebra at the beginning of middle school did not break down the stability of growth across mathematical areas (as seen in Table 1). Recall that accelerated students grew faster than non-accelerated students regardless of their initial mathematics achievement (see Table 3). Consistent correlations under such a condition imply clearly that accelerated students upgraded their ranks in growth in the same way simultaneously across all mathematical areas. In this sense, early acceleration of students into formal algebra at the beginning of middle school did promote stability of growth across mathematical areas.

The present study also used standardized coefficients associated with status of early acceleration across four mathematical areas as another measure of stability of growth across mathematical areas, with the analytical logic of research remaining the same. Overall, although early acceleration of students into formal algebra at the beginning of middle school was found to be related with consistent changes in rank in rates of growth, standardized coefficients indicate that the amount of addition to growth resulting from early acceleration varied somewhat. Early acceleration added somewhat more growth to algebra and geometry than to basic skills and quantitative literacy.

4.4. Do student and school characteristics moderate stability of growth?

The analytical logic of research in the present study is that correlations of rates of growth across mathematical areas among students and schools with adjustment for early acceleration of students into formal algebra at the beginning of middle school were used as the base line data (see Table 2) against which correlations further adjusted for the effects of student and school characteristics were compared. A comparison of Tables 4 and 5 to Table 2 shows that student and school characteristics did not make much difference in stability of growth across mathematical areas. The largest change in correlation between Tables 2 and 5 was 0.01 among students and 0.05 among schools. Even the largest, isolated correlation of 0.13 (at the school level) fall short of being practically important. Therefore, the upgrading in rank in terms of rates of growth of accelerated students (consistent across mathematical areas as discussed earlier) was not much influenced by student and school characteristics.

Finally, standardized coefficients associated with status of early acceleration (used as another measure of stability of growth across the four mathematical areas) were quite similar across Tables 2, 4, and 5. These findings indicate that student and school characteristics together did not influence much the amount of addition to growth resulting from early acceleration of students into formal algebra at the beginning of middle school. Specifically, the phenomenon that early acceleration added somewhat more growth to algebra and geometry than to basic skills and quantitative literacy was not much influenced by student and school characteristics.

4.5. Limitations of the study

Secondary data analysis often inherits limitations of original data. The LSAY data have good measures of school characteristics in terms of school context (including teacher characteristics) and school climate. These measures were used in the present study to adjust for school effects. However, the LSAY data do not have adequate measures on school curricular and instructional characteristics under which students make progress in mathematics. As a result, the present study could not take into account how well formal algebra was taught to accelerated students. Such a lack emphasizes that findings in the present study mark only the beginning of a comprehensive investigation into the issue of early acceleration of students in mathematics.

With a four-level structure, HLM models in the present study were already considerably complex. Coupled with a relatively small sample size in the LSAY, some more advanced data analyses could not be performed. For example, it would be insightful if higher order trends in growth could be explored to identify, say, whether students who were accelerated into formal algebra tended to experience a rapid growth in middle school and then a slow growth in high school. Further studies may seek more efficient statistical models or larger longitudinal samples to explore higher order trends.

Nevertheless, as a starting point along this line of research, the present study has offered important theoretical and practical insights. Specifically, early acceleration of low achieving students into formal algebra may be an effective strategy to improve mathematics achievement, and all low achieving students may have equal potentials to take advantage of early acceleration into formal algebra regardless of their individual, family, and school characteristics. Early acceleration may also maintain stability of growth across mathematical areas, and this stability may not depend on student and school characteristics. With certain limitations, the present study is also an invitation for more refined and advanced research on early acceleration of students in mathematics as it relates to their cognitive development.

Appendix A. Description of school climate variables

A.1. Computer-student ratio in mathematics

This variable measures how many students in a school share one computer that has been allocated especially for mathematics instruction.

A.2. Academic press

This equally weighted, aggregated measure (from the teacher level to the school level) includes items that describe teachers' impressions of a school's academic environment: (a) the learning environment in this school is not conductive to school achievement for most students; (b) staff members maintain high standards of performance; (c) most students in this school work up to their ability; (d) the teachers in this school push the students pretty hard in their academic subjects; and (e) in this school, there is really very little a teacher can do to insure that all of his/her students achieve at a high level (1, disagree strongly; 2, disagree somewhat; 3, disagree; 4, agree; 5, agree somewhat; 6, agree strongly).

A.3. Disciplinary climate

This equally weighted, aggregated measure (from the teacher level to the school level) includes items that ask teachers to describe the extent to which each of the following is a problem in their school: (a) students absenteeism from class; (b) robbery, theft, or vandalism; (c) student use of drugs or alcohol; (d) fighting or assault; and (e) verbal abuse of teachers (1, serious; 2, moderate; 3, minor; 4, none).

A.4. Parental involvement

Parents are asked to indicate whether, during this school year, they have done any volunteer work in their children's school. Parents' responses are aggregated to the school level. This aggregated response has been standardized with a mean of zero and a standard deviation of one. Parental involvement, therefore, measures parent volunteer work for school in the current study.

A.5. Principal leadership

This equally weighted, aggregated measure (from the teacher level to the school level) includes items that register teachers' opinions about their principals: (a) the principal deals effectively with pressures from outside the school that might interfere with my teaching; (b) the principal sets priorities, makes plans, and sees that they are carried out; (c) staff are involved in making decisions that affect them; (d) the school administration's behavior toward the staff is supportive and encouraging; (e) the principal seldom consults with staff members before he/she makes decisions that affect us; (f) goals and priorities for the school are clear; and (g) the principal lets staff know what is expected of them (1, disagree strongly; 2, disagree somewhat; 3, disagree; 4, agree; 5, agree somewhat; 6, agree strongly).

A.6. Teacher autonomy

This variable is the sum of equally weighted school aggregates (from the teacher level to the school level) of two composite measures. The first component includes items that ask teachers about how much influence they have in their school over policy in each of the following areas: (a) determining student behavior codes, (b) determining the content of in-service programs, (c) setting policy on grouping students in classes by ability, and (d) establishing the school curriculum (1, none; 2, minor; 3, some; 4, moderate; 5, much; 6, a great deal).

The second component includes items that ask teachers about how much control they feel they have in their classroom over each of the following areas of planning and teaching: (a) selecting textbooks and other instructional materials; (b) selecting content, topics, and skills to be taught; (c) selecting teaching techniques; (d) discipline students; and (e) determining the amount of homework to be assigned (1, none; 2, minor control; 3, some control; 4, moderate control; 5, much control; 6, complete control).

A.7. Teacher commitment

This equally weighted, aggregated measure (from the teacher level to the school level) includes items that measure teachers' attitudes toward their profession: (a) there is a great deal of cooperative effort among staff; (b) I usually look forward to each working day at this school; (c) I sometimes feel it is a waste of time to try to do my best as a teacher; (d) I am familiar with the content and specific goals of the courses taught by other teachers in my department (1, disagree strongly; 2, disagree somewhat; 3, disagree; 4, agree; 5, agree somewhat; 6, agree strongly).

A.8. Mathematics homework

Teachers are asked to indicate how many hours of mathematics homework they assign to students in a typical week. An aggregate is created to measure the average hours of mathematics homework that teachers in a school assign to their students in a typical week, used as a continuous variable.

A.9. General support for mathematics

This equally weighted, aggregated measure (from the teacher level to the school level) includes items that register teachers' perceptions on the extent to which each of the following is a problem in their school: (a) some teachers are inadequately trained to teach mathematics, (b) lack of teacher planning time, (c) class sizes too large, (d) students with different abilities and interests taking the same mathematics classes, (e) too little coordination or articulation between classes in the mathematics curriculum, and (f) too few advanced mathematics courses in the curriculum (1, serious; 2, moderate; 3, minor; 4, none).

A.10. Extracurricular activities

This is a measure that taps how many extracurricular activities that a school offers to their students: (a) science club, (b) mathematics club, (c) computer club, (d) jets, and (e) engineering/technology club. For example, the dummy variable, science club, is coded as 1 if schools offer a science club, 0 if schools do not. A total score is calculated that sums over the 5 items for each school.

References

Alexander, P. A. (1997). Mapping the multidimensional nature of domain learning: The interplay of cognitive, motivational, and strategic forces. In P. R. Pintrich & M. L. Maehr (Eds.). *Advances in motivation and achievement* (Vol. 10, pp. 213–250). Greenwich: CT: JAI.

Alexander, P. A. (2003). The development of expertise: The journey from acclimation to proficiency. *Educational Researcher, 32*, 10–14.

Boero, P., Douek, N., & Ferrari, P. L. (2002). Developing mastery of natural language: Approaches to theoretical aspects of mathematics. In L. D. English (Ed.), *Handbook of international research in mathematics education* (pp. 241–270). Mahwah, NJ: Lawrence Erlbaum.

Crocker, L., & Algina, J. (1986). *Introduction to classical and modern test theory*. New York: Holt, Rinehart, & Winston.

Frevost, F. J. (1985). Soundoff: Let's not teach algebra to eighth graders! *Mathematics Teacher, 78*, 586–587.

Karper, J., & Melnick, S. A. (1993). The effectiveness of team accelerated instruction on high achievers in mathematics. *Journal of Instructional Psychology, 20*, 49–54.

Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390–419). New York: Macmillan.

Kleiner, I. (1989). Evolution of the function concept: A brief survey. *College Mathematics Journal, 20*, 282–300.

Kolitch, E. R., & Brody, L. E. (1992). Mathematics acceleration of highly talented students: An evaluation. *Gifted Child Quarterly, 36*, 78–86.

Liu, Z., & Liu, P. (1997). Practice and theoretical exploration on supernormal education of mathematics to gifted and talented children from6-8. In J. Chan, R. Li, & J. Spinks (Eds.), *Maximizing potential: Lengthening and strengthening our stride. Proceedings of the 11th World Conference on Gifted and Talented Children* (pp. 130–135). Hong Kong: University of Hong Kong.

Miller, J. D., Kimmel, L., Hoffer, T. B., & Nelson, C. (2000). *Longitudinal study of american youth: Users manual*. Chicago, IL: International Center for the Advancement of Scientific Literacy, Northwestern University.

Mills, C. J., Ablard, K. E., & Gustin, W. C. (1994). Academically talented students' achievement in a flexibly paced mathematics program. *Journal for Research in Mathematics Education, 25*, 495–511.

Oakes, J. (1990). *Multiplying inequalities: The effects of race, social class, and tracking on opportunities to learn mathematics and science*. Santa Monica, CA: Rand.

Rasbash, J., Browne, W., Goldstein, H., Yang, M., Plewis, I., Healy, M., Woodhouse, G., Draper, D., Langford, I., & Lewis, T. (2000). *A user's guide to MLwiN*. University of London.

Raudenbush, S. W., & Bryk, A. S. (2002). *Hierarchical linear models* (2nd ed.). Newbury Park, CA: Sage.

Raudenbush, S. W., Rowan, B., & Kang, S. J. (1991). A multivariate, multilevel model for studying school climate with estimation via the EM algorithm and application to US high-school data. *Journal of Educational Statistics, 16*, 295–330.

Rogosa, D. R., Brandt, D., & Zimowski, M. (1982). A growth curve approach to the measurement of change. *Psychological Bulletin, 90*, 726–748.

Smith, J. B. (1996). Does an extra year make any difference? The impact of early access to algebra on long-term gains in mathematics attainment. *Educational Evaluation and Policy Analysis, 18*, 141–153.

Spring, J. (1989). *The sorting machine revisited: National educational policy since 1945*. New York: Longman.

Teddlie, C., & Reynolds, D. (Eds.). (2000). *The international handbook of school effectiveness research*. London: Falmer.

Useem, E. L. (1993). *Renewing schools: A report on the cluster initiative in Philadelphia*. Philadelphia, PA: Paths and Prism.

Usiskin, Z. (1987). Why elementary algebra can, should and must be an eighth-grade course for average students. *Mathematics Teacher, 80*, 428–438.

Usiskin, Z. (1995). Why is algebra important to learn?. *American Educator, 19*, 30–37.

Wagner, S., & Kieran, C. (Eds.). (1989). *Research issues in the learning and teaching of algebra*. Hillsdale, NJ: Lawrence Erlbaum.