High School Students’ Epistemological Views of Mathematics

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Abstract

Past and current education reform documents describe a guiding mathematics curriculum that promotes the understanding of the subject of mathematics in terms of three components: content, process, and a way of knowing. While mathematical content and processes have been foundational in the mathematics curriculum, little is known about students' capabilities in understanding mathematics as a way of knowing. This interpretive study examined the views of n = 39 high school students with particular attention to their epistemological beliefs about mathematics. Findings from this study indicated that most students viewed all mathematical problems stemming from real-world problems and that mathematical justification was solely empirical. Implications for the influence of real-life problems in the mathematics classroom on students' epistemological beliefs include 1) curricular changes that foster solving problems that are not necessarily connected to the real-world 2) the need the address mathematical epistemology in the classroom

Introduction

Past and current education reform documents describe a guiding mathematics curriculum that promotes the understanding of the subject of mathematics in terms of three components: content, process, and a way of knowing (American Association for the Advancement of Science [AAAS], 1990; National Council of Teachers of Mathematics [NCTM], 1989, draft). Schwab (1978) referred to the first two components, content and process knowledge, as venues for describing the structure of any discipline. Content, or substantive knowledge, consists of the "conceptual devices" which are necessary for determining the boundaries of the discipline and are used to create new content knowledge within the discipline. Quantification, equality, and parallelism are all examples of conceptual devices used in mathematics used as building blocks in the development of new mathematical knowledge. Process, or syntactical knowledge, refers to the "logical structures" of a discipline used for verification or justification for new knowledge. Both inductive and deductive reasoning are used for verification in mathematics while logical deductive proof has been the accepted form of justification for centuries.

The third component, mathematics as a way of knowing, refers to the salient characteristics or features of mathematics that describe the substantive and syntactical knowledge of the discipline of mathematics. Mathematical content and process knowledge focus on "doing" mathematics. Mathematics as a way of knowing refers to understanding "about" mathematics. There is a significant difference between doing mathematics and understanding about mathematics.

While not all philosophers and mathematicians agree on the defining features of mathematics, this third component, commonly referred to as the nature of mathematics, has been defined explicitly in Science for All Americans (AAAS, 1990) and implicitly in Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989). The two reform documents differ slightly in semantics, however, both can be linked to the educational psychology literature regarding the nature of knowledge (See Table 1).

Table 1: Overview of the Nature of Mathematics

Epistemological Constructs in Educational Psychology	***Science for All Americans*** **(AAAS, 1990)**	***Curriculum and Evaluation Standards for School Mathematics*** **(NCTM, 1989)**
Certainty (Perry, 1970)	• Mathematical insights into abstractions have grown over thousands of years, and they are still being extended—and sometimes revised (p. 20).	(not defined)
Source (Perry, 1970)	• …an interplay between imagination and rigorous logic; ideals of honesty and openness; the critical importance of peer criticism (p. 18).	• Mathematics is a process, a body of knowledge, and a human creation (p. 128).
Simplicity (King, Kitchener, Davison, Parker, & Wood, 1983)	• As mathematics has progressed, more and more relationships have been found between parts of it that have been developed separately – for example, between symbolic representations of algebra and the spatial representations of geometry (p. 16). • These cross-connections enable insights to be developed into the various parts; together, they strengthen belief in the correctness and underlying unity of the whole structure (p. 16).	• Students' understanding of the connections among mathematical ideas facilitates their ability to formulate and deductively verify conjectures across topics (p. 147)
Justification (King, Kitchener, Davison, Parker, & Wood, 1983)	• A central line of investigation in theoretical mathematics is identifying in each field of study a small set of basic ideas and rules from which all other interesting ideas and rules in that field (p. 16). • Mathematics explores possible relationships among abstractions without concern whether those abstractions have counterparts in the real world (p. 16).	• ….the requirement to verify statements with a deductive proof by reasoning from axioms is unique to the discipline of mathematics (p. 145). • Students at all levels must develop an intuitive sense that mathematics is based on established rules and is not a 'bag of tricks' familiar only to those who teach or develop mathematics. It is particularly important that advanced students understand that there is an element of arbitrariness in how the rules are selected but that the encompassing system is consistent" (p. 221).

Much of the foundational theoretical framework of epistemological beliefs is accredited to Perry (1970). His work with Harvard students' views about the nature of knowledge established a springboard for epistemological constructs. Through conversations with the students, Perry (1970) examined students' beliefs about the certainty and source of knowledge. As college students described their views of knowledge, Perry (1970) developed a scheme of intellectual and ethical development based on a continuum from absolutist to relativist. Students at the absolutist stage would have difficulty understanding how competing scientific theories can exist while students at the relativist stage would be able to commit to a belief base on specific context. Certainty and source of knowledge can be naively described in terms of levels of tentativeness and subjectivity, respectively. Extending on Perry's (1970) work, King, Kitchener, Davison, Parker, and Wood (1983) identified developmental stages for the nature of knowing in terms of justification and simplicity of knowledge. The justification of knowledge refers to its method of acceptance by the student or, more generally, by a certain community of experts in the particular discipline. The simplicity of knowledge refers to the level of interconnectedness of the components of a particular body of knowledge: the less connected, the more simplistic.

Pursuits of understanding students' epistemological beliefs have recently been resurrected in studies conducted by Schommer (1993, 1995, 1997). The focus of her studies has been on the structure and development of epistemological beliefs and the influence of beliefs on learning. Through her studies, Schommer provided insight into high school students' beliefs and the notion of domain-dependent beliefs. In a study examining high school students' beliefs about knowledge, Schommer, Calvert, Gariglietti, and Bajaj (1997) suggested that high school students held beliefs about knowledge and also showed growth between ninth and twelfth grade. Schommer and Walker (1995) noted inconsistencies in some epistemological aspects of students' beliefs of general knowledge across two different disciplines. Thus, examining general beliefs does not provide enough evidence to generalize to domain specific beliefs.

Possible factors contributing to students' epistemological beliefs were investigated by Jehng, Johnson, and Anderson (1993). They suggested that epistemological beliefs were associated with the fields of study in which the students were engaged. The field of study was determined by the types of problems the students studied in their required classes. For example, "soft" science fields were fields of study that often investigated ill-formed problems while "hard" science fields did not. Students who studied "soft" sciences were more likely to view knowledge as absolute and simple as opposed to students who studied "hard" sciences. Students who studied "hard" sciences were more likely to view knowledge as passed down from authority. However, these findings were based on general characterizations of "soft" and "hard" science fields as opposed to observations of how the curriculum has been implemented in the classroom.

Students' views, specific to mathematical beliefs, were first investigated by Schoenfeld (1983) in an attempt to link belief systems and strategy selection during problem solving. In the study, two students were asked to make a geometric construction during a think-aloud activity. The students' comments suggested that they held a simplistic view of mathematical concepts whereby mathematical knowledge was viewed as a list of disconnected facts. In addition, their view of justification of their solution to the problem was based on empirical evidence (drawing by hand) rather than notions of proof by logical argumentation. Findings from that study suggested a link between students' views about the simplicity of mathematical knowledge and how they viewed mathematical justification. Viewing mathematical knowledge as disconnected, isolated facts was linked to viewing mathematical justification as empirical.

In recent years, Ruthven and Coe (1994) attempted to focus on high school students' epistemological beliefs about mathematics and about learning mathematics. With the use of a Likert questionnaire, these researchers attempted to examine the students' beliefs regarding certainty, source, and justification of mathematical knowledge as well as their beliefs about learning mathematics. One finding in their study suggested inconsistencies in students' beliefs about the source of mathematical knowledge, supporting neither absolutist nor fallibilist perceptions. The researchers found six factors regarding the students' beliefs about (personal) justification of mathematical knowledge and learning mathematics. The researchers proposed that if these factors represent six independent beliefs, students' beliefs about the justification of mathematical knowledge are rather complex. This complexity may imply greater difficulty for learning in the mathematics classroom.

While Ruthven and Coe's (1994) study provided insight into the structure of students' epistemological beliefs, use of a Likert-type questionnaire without follow-up interviews may have provided incomplete findings. Lederman and O'Malley (1989) showed, through interviewing, high school students' meanings of such concepts as theory and law were varied and often incongruent with respect to the current definitions in the science education community. It is likely that definitions of mathematical terminology used in their instrument (such as "proof" and "valid") could have potentially a variety of meanings for the students.

The purpose of this study was to gain insights into the depth and breadth of high school students' views about mathematics as a way of knowing using an interpretive approach. The major focus of the study was guided by the following question: What are high school students' epistemological views about mathematics?

Subjects

This study was conducted at a small, rural high school in the Pacific Northwest. All 39 of the students who volunteered to participate in the study were enrolled in one of three mathematics classes, Geometry (22 students), Algebra II (11 students), or Pre-Calculus (6 students), all being taught by the same teacher. Four students (1 Geometry, 1 Pre-calculus, and 2 Algebra II) chose not to participate in the study. The classes were heterogeneous in ability. However, Algebra II and Pre-calculus classes were considered elective mathematics classes. The students (24 males, 15 females) chose their own pseudonym in order to maintain anonymity.

Method

The methods for this interpretive study were selected to elicit and validate students' views about mathematics without the imposition of 'a priori' epistemological constructs. Since this study involved potential multiple meanings of common mathematical terms, the method for this study included the administration of an open-ended questionnaire followed by an interview. Students were given a written questionnaire consisting of the following:

1. What, in your view, is mathematics? Feel free to make a list or diagram to help you express your ideas about mathematics.
2. If you were a mathematician, what would you be doing when you worked?

Five university professors validated the questions (1 mathematics educator, 1 mathematics professor, 1 psychology professor, and 2 science educators). Suggestions from this panel for the original questionnaire included the deletion of a third question and rewording of a second question to match the intellectual developmental level of high school students. Face and content validity of 100% for each question were obtained. Both questions were designed to provide students with the broadest context for talking about mathematics. In addition to clarifying and validating students' responses through the questionnaire and interview procedure, the method allowed students to provide individual perspectives about their beliefs according to their own background with mathematics.

The classroom teacher administered the questionnaire to the students during their mathematics classes. Sixteen participants (6 Geometry, 4 Algebra II, and 6 Pre-Calculus) were chosen for a single follow-up interview in order to probe students' initial responses to the questionnaire. Interviews were conducted in conversational style (Mishler, 1986) by one of the researchers. Audio tapes were transcribed and coded by the same researcher. General interview probes were developed to enable students to clarify the meaning of their responses and to provide students with opportunities to validate their responses through examples and explanations. In addition to the questionnaire and interviews, the interviewer kept a journal to record the students' level of commitment about their beliefs.

Data analyses were guided by the main research question regarding students' epistemological views. The responses from the questionnaire were analyzed in the order that they were presented to the students. After the questionnaires were read and examined for similarities and differences, the students were selected for the follow-up interviews. Selection of the students for interviews was based on their grade level and on the similarities and differences of their responses in order to investigates a broad range of views. In addition, students' attitudes toward mathematics were disregarded in the analyses. The similarities and differences were categorized into three main categories: mathematical knowledge, mathematical processes, and mathematical ways of knowing. Finally, responses in the last category, mathematical ways of knowing, were divided into the emerging subcategories.

Results

Trends in the data analysis indicated that Algebra II and Pre-calculus students tended to discuss mathematics across all three main categories while Geometry students tended to focus mainly on mathematical processes.

Mathematical Knowledge. This category consisted of words or phrases used by the students that described mathematics in terms of mathematical "objects." Students referred to a variety of mathematical objects: numbers, shapes, variables, formulas, equations, properties, postulates, and theorems. "Number" was the most frequently mentioned object from the questionnaire.

> When I think of mathematics, I usually think of numbers. Complex or real. The first thing I think of is numbers. When I think of mathematics more, I realize that its not just numbers. In Pre-calculus, for example, we deal a lot with letters and story problems. (Eli, Pre-calculus)

Several students interchanged the terms *postulate* and *theorem* and realized that there was a distinction between the terms that they were unable to describe. On the contrary, students interchanged formula and equation but were not aware of any difference between the two terms.

Mathematical Processes. This category consisted of words or phrases used by the students that described "doing" mathematics. Students referred to the following words or phrases: solving, calculating, formulating, communicating, explaining, testing, experimenting, proving, checking, assuming things true, convincing, and generalizing.

> What mathematics really is, is calculating numbers and solving complex equations using mathematical skills. (P16, Algebra II)

Interestingly, a few students believed that guessing was not a mathematical way to solve problems such as this Pre-calculus student:

> Vince:
> In my view, mathematics is solving problems scientifically with numbers.
>
> Interviewer:
> What do you mean by that?
>
> Vince:
> Use exact things. You follow certain rules and it's not guessed. Use real formulas, not just guess and check.

Mathematical Ways of Knowing. This category consisted of characteristics or features of mathematical knowledge, emerging as seven epistemological subcategories: connectedness, tentativeness, human endeavor, abstractness, efficiency, arbitrariness, and non-empirical. Approximately half of the students described at least one feature of mathematics as a way of knowing. Their views represented a continuum of beliefs (see Table 2). For the most part, the students seemed to possess a solid commitment in the areas of connectedness and tentativeness of knowledge. However, many students expressed uncertainty about their understanding of how mathematicians know whether they have solved a problem or proved a theorem. In addition, some of the students said that mathematicians make up their own problems to investigate, but also seemed uncomfortable in verbalizing human invention of these mathematical problems and theorems.

Table 2: Frequency of student remarks on emerging features of mathematics by grade level (n=39)

Epistemological Feature	Geometry	Algebra II	Pre-calculus	Total
Connectedness				
Disconnected	0	1	1	2
Interrelated	3	1	1	5
Tentativeness				
Static	0	0	0	0
Dynamic	0	0	1	1
Tentative	1	1	0	2
Human Endeavor				
Personal benefit	10	1	2	13
Human advancement	1	1	1	3
Abstractness				
Concrete	3	0	0	3
Abstract	0	1	0	1
Efficiency	1	2	2	5
Arbitrariness				
Facts	0	0	1	1
Opinion/Assumptions	3	2	2	7
Non-Empirical				
Empirical	2	1	3	6
Non-Empirical	2	0	0	2

Connectedness

Of the 39 students surveyed, seven students commented on whether mathematics consists of interrelated concepts. Five of these students explicitly referred to mathematics as a body of interrelated knowledge. The students either referred to the "wholeness" of the mathematics or described relationships between "parts" or "branches" of mathematical knowledge such as numbers and shapes:

> Mathematics is system that uses numbers, letters, and symbols to explain different shapes and other things that occur in everyday life. (Ben, Geometry)

> Mathematics to me is anything that has to do with numbers and all the different fields of mathematics stem from this. (Bob, Algebra II)

The two other students did not view mathematics as a connected body of knowledge. Both referred to mathematics as a fact-based discipline. In a discussion about numbers as facts, this Algebra II student was not able to establish the role of imaginary numbers in the number system:

> Interviewer:
> Would you say that a number is a fact?
>
> Cheshire:
> Sure (pause) but then I wouldn't consider imaginary numbers as facts.
>
> Interviewer:
> What are they?
>
> Cheshire:
> They're *i*. Like the negative square root is *i*. That's an imaginary number.
>
> Interviewer:
> What would you call it if it is not a fact?
>
> Cheshire:
> Somewhere out in space.

<u>Tentativeness</u>

Only two of the 39 students, Greg and Paladine, remarked about the tentatitveness of mathematical knowledge. Greg held a dynamic view of mathematics. He was confident that mathematicians will create new knowledge and that prior knowledge is unchanging. He viewed mathematics as a changing discipline, but only because it is expanding. Paladine held a tentative view of mathematics and supported this views by an explanation involving the creation of imaginary numbers and fractals. He also supplied an example about the possibility of defining the division of zero differently to create another branch of mathematics. His view of tentativeness in mathematics was expressed as an abrupt paradigm shift in thinking:

> I don't think that I should have put 'changing.' The laws and principles usually don't change. But as far as the new stuff…it is getting bigger, yes, getting larger. (Greg, Pre-Calculus)

> It can (change). I don't think that it will, but I think that it mostly is going to build off of what it has and its not going to change the basis of that, unless something drastic happens. (Paladine, Algebra II)

<u>Human Endeavor</u>

Almost half of the 39 students commented on the purpose of mathematics as relating to the human endeavor. Two perspectives of mathematics as a human endeavor emerged: personal benefit and human advancement. Most of the students who remarked about the purpose of mathematics held the perspective that mathematics is pervasive in their own daily lives. Students viewed this pervasiveness mainly in terms of necessary basic mathematical skills for solving everyday problems.

> Many people don't realize how much they use math everyday. We use math skills when we buy objects, estimating distances, solving problems, and even watching the clock. (Maple Belle, Geometry)

Two students believed that mathematics was pervasive but only in terms of its presence in the world around them and not for direct personal benefit.

A few students held the view that mathematics was part of the human endeavor as a tool created for human advancement. They believed that mathematics was necessary for scientific and technological advancement:

> Math is a tool and without it the world would still be cavemen and rock clubs. Math is a burden to some but without it there would be a burden for all. (Stimpy, Geometry)

Abstractness

A few students used the term "created" in their description of mathematics. However, when posed with the alternative view of discovery, all of the students said that mathematics was discovered as well as created. Only one student indicated that mathematics was "man's first attempt at abstract thought":

> Math does not exist. It is all in our heads…There is a way of applying it. Math is generally abstract. You can take math and apply it to things, like apples and oranges, so it can be applied to real life, non-abstractly…It's all abstract. (Paladine, Algebra II)

However, all of the other students viewed mathematics as rather concrete and tied to the physical world. One student remarked that all of the problems in his text were problems that someone would come across in real life. Stimpy's view seemed typical of the group of students who were committed to the belief that mathematics is solely connected to the physical world.

> I see math and it's an answer to something. And if it wasn't physical, you couldn't show that it was an answer to something…then it really wouldn't be math. It would be something else. (Stimpy, Geometry)

Efficiency

In describing the work of a mathematician, five students remarked about finding multiple solutions to problems and about finding the "easiest" solution to a problem. All of the students indicated that finding different solutions to problems was an act of efficiency rather than elegance.

> I would be working on new formulas so it would be easier for students and other people to solve difficult problems. (Travis, Algebra II)

Arbitrariness

Several students remarked about the acceptance of postulates and definitions. However, when given the opportunity to talk further about postulates and definitions, many only commented nonchalantly about accepting mathematical truths. Only a few of these students seemed to realize the subjective nature of the axiomatic system:

> You are basing it all on someone else's opinion. So, who is to say that he's not wrong when he says 1 + 1 = 2. Who is to say that is not right? Everyone just accepts it…so it could be wrong. (Flora, Pre-Calculus)

> To judge stuff. To give it names, like definitions, or how big stuff is…Before we had math, we could say it was big and somebody else could say that it was bigger. There was no way to prove it. You could look at it but you wouldn't have any sense of height because there is nothing to judge it by. (Curt, Geometry)

Only one student expressed the opposite view held by Flora and Curt. She believed that mathematics was absolute and non-arbitrary:

> It's all facts. You are given numbers and numbers don't have opinions. (Mary Margaret, Pre-Calculus)

Non-Empirical

It seemed virtually impossible for most students to separate mathematics and science in their discussions. For example, one of those students held the view of mathematical justification as a facsimile of scientific justification, the observation of numbers in a mathematical experiment:

> "Testing. Trying to solve them to see if this works or something else works. Testing a postulate. They (mathematicians) do experiments to see if they are true…It [math experiments] deals with numbers and those sorts of things. Scientists use other things than mathematicians." (Shmoopy, Geometry)

More typical were the responses that linked mathematical modeling as the job of a mathematician. In addition students generally described the work of a mathematician as solving complex equations rather than producing new knowledge through proof. Thus, students discussed an empirical view of mathematical justification. These students discussed the value of observation and "double checking" the results:

> You would have to trial it…And they sometimes actually can use math and double check. Just like a math check. (Stimpy, Geometry)

> I guess the only way you could tell is if you test it. Use your formula on something…just trying to find another way to check it. (Vince, Pre-Calculus)

During the interviews, several students interchanged solving and proving, indicating solving was synonymous with proving. Six students were questioned about the meaning of the two terms. Four of the students indicated synonymous meanings. All four went on to define solving and proving as a way to explain the steps in the problem. The other two distinguished proving as a way of convincing in addition to explaining. Thus, for some, proving was viewed as an empirical and algorithmic action:

> You are given a problem and you have to solve it. You have to have reasons why…direct reasons, like theorems…You know your answer…It's like you are in one spot in time and you try to figure out how to get to the next spot in time and you have to have your reasons. (Mary Margaret, Pre-calculus)

> [Proving] by mathematically going through the steps…You can go through different steps using different postulates to prove that those two sides of a triangle are equal. (Cheshire, Algebra II)

Mathematicians. Only a few students were unable to place themselves in the position of a mathematician. Students referred to both mathematical activities and specific job titles in their description of mathematicians. Approximately three-fourths of the students referred to mathematical activity, which included solving, proving, and discovering (included in Table 2). Almost one-half of the students (mostly Geometry students) cited teaching as the main job of a mathematician. Ten students referred to jobs that required mathematical skills such as research in a science laboratory, construction and architecture, decoders for the military, and accounting. Students also viewed mathematicians as people who can do almost any job.

> If I were a mathematician, I would hire myself out to do lots of different jobs from construction to building airplanes. There are so many things that a mathematician could do they could go into anything they wanted. (Bob, Algebra II).

One major inconsistency existed in a comparison between students' views of mathematics and what they think mathematicians do. Many students mentioned proof as a feature of mathematics, yet did not mention that proving was an activity for mathematicians.

Discussion

This study revealed three major findings. The first finding was that the high school students in this study demonstrated an intellectual capability for the nature of mathematics. Over half of the students in this study held beliefs, at least initial ones, about the nature of mathematics, indicating that intellectual capability of this type is not limited to a just a few students. This finding supports curricular goals for the nature of mathematics expressed in the NCTM Standards as an educational outcome. The expectation for addressing the nature of mathematics in high school classrooms can not be realized if students are not intellectually capable of thinking about mathematics as a way of knowing. Moreover, this finding also suggests that high school teachers should expect that students in their classrooms have most likely already started to develop beliefs about the nature of mathematics. Thus, high school teachers need to hold adequate conceptions of the nature of mathematics and be able to address it in their own classrooms.

This implication raises a question about the research focus on teachers conceptions of mathematics which is currently directed toward understanding the relationship between teachers' conceptions of mathematics and their behavior in the classroom (Cooney, Shealy, & Arvold,1998; Thompson, 1984; Raymond, 1997). Research on teachers' beliefs should be directed toward developing adequate conceptions of mathematics and in terms of a broader range of epistemological constructs to include the ones discussed in this article. Related to this issue is the development of teaching strategies for the nature of mathematics.

Another important finding was that a continuum of views was expressed by the students across the various emergent features. The continuum represented views that were both consistent and inconsistent with reform documents. The most common misconception expressed by the students was the idea that mathematical knowledge is justified only through observation of physical phenomena. Their views of mathematicians as "users" rather than "makers" of mathematics corroborated the students' view of mathematics as empirical.

One possible explanation for this finding might be the increased attention to real-world problems in the mathematics curriculum may mislead students to thinking that mathematics must be linked to the real-world. During an interview, one student indicated that his textbook did not contain any problems that were purely mathematical. Although based on his perception of his mathematics classes, the use of real-world problems seems to be consistent with the current trend in the mathematics curriculum to make mathematics more relevant. Real-world problems may motivate students, but its influence in the curriculum may provide an incomplete picture of how mathematical knowledge is justified. An overuse of real-world problems in the curriculum might also account for students' views that mathematicians can do anything. While it is true that mathematics can be connected to a variety of jobs, students should realize that the job of a mathematician is quite different from that of an engineer.

Students also may be forming a distorted view of applied mathematics, a discipline that does not exclude proof as a way of justifying knowledge. Unfortunately the curriculum (textbooks, teaching, additional resources) was not investigated in this study. A closer examination of the possible influence of real-world problems on students' beliefs about mathematics as a way of knowing is needed.

The implications of this finding are important for future mathematics curricula as well as for curricula that integrate mathematics. If the purpose for integration of subject matter is to promote a seamless entity, then justification of mathematical knowledge, which is different from other disciplines, then how is justification of knowledge (scientific or mathematical) addressed in an integrated curriculum? Interdisciplinary curriculums, which distinguish branches of knowledge, would be a more fruitful curriculum endeavor.

Another feature of justification of mathematical knowledge viewed by the students that was problematic was arbitrariness. Students also referred to the arbitrariness of mathematics in terms of "things that couldn't be proved." Students were unable to discuss this feature in detail and seemed to treat this feature with unimportance. In addition, students did not connect this idea to mathematics as a system, it seemed doubtful that students truly understood this feature.

The third finding was that none of the students thought that mathematics involved creativity or imagination. One possible explanation for the absence this feature is that the students seemed to be more concerned with the efficiency of mathematics. Perhaps the existence of algorithms in mathematics gives students a false impression of the nature of mathematics. Even though creativity is used in developing algorithms, student may only be aware of the application of algorithms.

In conclusion, further study in this area is needed to better understand students' views of the nature of mathematics, and the relationships, if any, between students' and learning and doing mathematics. For example, what factors in the classroom are involved in the formation of students' views about the justification of mathematics? Curriculum and teaching methods were discussed in this study as two possible factors based on students' comments regarding the types of problems encountered in their mathematics classroom. A more thorough investigation of mathematics curricula would be needed to get a more complete picture of the reasons why these students viewed mathematics as an empirical-based discipline. Specifically, what is the role of real-world and pure mathematical problems in the classroom? And, how do teachers address mathematical justification in the classroom?

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