INTEGRATING SCIENCE AND MATHEMATICS: A NATURAL CONNECTION OR STRANGE BEDFELLOWS?

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Introduction

Integration of science, mathematics, and other disciplines is advocated in current national reform efforts (NCTM, 2000; NRC, 1996; Rutherford & Ahlgren, 1990). *Science for All Americans* suggests that science and mathematics have many similarities that make integration of the disciplines natural: “Mathematics is the science of patterns and relationships. Mathematics is also an applied science. Mathematics is the chief language of science” (Rutherford & Ahlgren, 1990, pp. 16-18). These claims beg the question, “Is mathematics a science?”

True integration has been defined as a seamless blending of disciplines so that distinctions between them are not evident (Lederman & Niess, 1997). Curriculum integration, some suggest, serves to not only increase student interest but also may help students develop abilities to think critically and make connections among disciplines in a manner useful and relevant to students (as discussed in Czerniak, Weber, Sandmann, & Ahern, 1999). However, research on the effectiveness of integration toward these claims is insufficient (Czerniak et al., 1999: Lederman & Niess, 1997; Pang & Good, 2000). In a recent review of the literature on integration of science and mathematics, Pang and Good (2000) discussed an endorsement for integration stemming from the view that scientific inquiry and mathematical problem solving are similar processes that serve similar purposes. Science and mathematics do seem to share several process skills such as searching for patterns, making conjectures, and reasoning as well as specific mathematical content. However, there are fundamental differences between science and mathematics that make true integration of the disciplines problematic (Lederman & Niess, 1998). The prominent difference between understanding the two disciplines occurs at the formal level of understanding and within the nature of the disciplines. Is mathematics a science? Exploring the natures of science and mathematics will serve to address this question. This paper will examine the natures of science and mathematics and their relationship to each other, as well as the implications of the relationship for the integration of science and mathematics.

Levels of Understanding

What does it means to "understand" a discipline? If integration is to lead to better understanding and application of knowledge, then "understanding" needs to be described. Throughout the past several decades, educational researchers have attempted to model what it means to understand in mathematics (Davis, 1979; Herscovics & Bergeron, 1983; Hiebert, 1986; Skemp, 1971, 1979; and Pirie & Kieren, 1994). Among the various models proposed, understanding has generally been portrayed as a dynamic element in the learning process as demonstrated in Skemp's (1971) definition: "to understand something is to assimilate it into an appropriate schema." Thus, the student's level of understanding is dependent on whatever schema or "conceptual structure" is deemed appropriate during classroom instruction. Three kinds of understanding were created to account for various schemas: instrumental, relational, and formal. In recent reform documents, all of these levels of understanding have been established as important components for teaching and learning content, process skills, and the natures of science and mathematics (National Council or Teachers of Mathematics [NCTM], 2000; National Research Council [NRC], 1996). While instrumental and relational understanding in science and mathematics classrooms seem remarkably similar, it is at the formal level of understanding that learning within the disciplines is distinguishable.

Instrumental understanding

Instrumental understanding refers to the kind of understanding achieved when students focus solely on the procedures used to produce correct answers. This does not necessarily mean that students have learned the procedures through rote memorization. Students may develop specific conceptual links between the procedures and the algorithm used to solve the problem. For example, students may be able to solve problems involving the geometric mean by linking the structure or form of the diagrams to equations used to find a missing side of a particular triangle within diagrams. However, the defining feature of instrumental understanding is that the procedures used to solve these kinds of problems would *not* be linked to the geometric concept of similarity. Similarly in science classrooms, instrumental understanding is demonstrated when students simply follow prescribed procedures to reach a conclusion. “The Scientific Method” is often presented and used in the classroom as such an algorithm. This approach, although valid in science, is not representative of all scientific investigations and serves to reduce the students’ conceptual understanding of scientific inquiry. Unfortunately, much classroom-based laboratory work has followed such an approach. As such, students’ understanding of scientific inquiry is held to this instrumental level. It should also be noted that strictly attending to one single method to scientific investigations does not portray an accurate image of scientific procedures.

Relational understanding

When student focus on establishing connections between the procedures and the concepts involved, then students develop relational understanding. Having achieved relational understanding, students are able to understand why the concept of similarity allows for certain equalities to be established from a given diagram involving similar triangles, yet students may not be able to carry out the procedures to produce the correct answers. In science, students may understand the importance of connecting conclusions to evidence or that valid inferences arise from empirical observations, yet these same students may not be able to design and conduct their own investigation. In short, relational understanding in science is demonstrated by an individual understanding about inquiry but not being able to "do" inquiry. Skemp (1979) suggested it is easy to determine whether students have achieved an instrumental understanding, but may be difficult to determine whether students have actually achieved a relational understanding. The connections the students make between the procedures and the concepts are often only known to that student and may not be easily articulated by that student.

Formal understanding

When students use their relational understanding in a public forum, the result is a display of formal understanding. In both science and mathematics classrooms, students use chains of logic to *explain* to others their reasoning process. However, when students use their relational understanding to *convince* others within a specific discipline, formal understanding in science and mathematics classrooms diverge.

Convincing others in the science classroom means that students draw inferences from empirical evidence (observations based in the real world) that support or refute a particular hypothesis. Further, connecting conclusions to available evidence in a logical and internally consistent way and being able to apply a given theoretical framework to support the conclusions, while recognizing valid alternatives, demands formal understanding of the discipline. In other words, one must have knowledge *about* inquiry and abilities to *conduct* inquiry within the context of science. In the mathematics classroom, mathematical proof is the socially acceptable vehicle for convincing others. Mathematical proof is based upon selected known axioms and theorems about non-empirical mathematical objects. In mathematics classrooms, students may test their conjectures by drawing inferences on a case-by-case basis but will ultimately try to convince others through mathematical proof. Deborah Tepper Haimo (1995) once related a tongue-in-cheek story about the differences between scientific and mathematical argumentation:

> “I remember that when I was a mathematics undergraduate many years ago, we used to consider physics majors as coming to conclusions that were invariably sloppy. Our favorite description of physicists, as distinguishable from mathematicians, was that they believed all odd numbers are prime, and that they reached that results by the argument that 3 is prime, 5 is prime, 7 is prime, 9 is an experimental error, 11 is prime, …q.e.d.”

It is also important to note that being able to draw inferences during scientific inquiry or to produce a mathematical proof does not guarantee that students have achieved a formal understanding. Students have achieved a formal understanding when they realize what it means to *justify* the knowledge within that particular discipline.

Nature of Science.

Nature of science (NOS) refers to the epistemology of science or science as a way of knowing. It is important to note that there is not one single nature of science, or “the NOS,” that fully describes all scientific knowledge and enterprises. However, there is agreement on certain aspects of NOS that are relevant and important to include in K-12 education (Lederman & Abd-El-Khalick, 1998). Chief among these is that scientific knowledge is tentative or subject to change and revision. The tentative NOS stems from several other NOS aspects, including the empirical basis of knowledge, subjectivity (observations and interpretations are theory-laden), creativity (science is the product of human imagination), and the sociocultural embeddedness of the knowledge. Such general elements of NOS are emphasized by national reform documents as important for achieving science literacy (AAAS, 1993; NRC, 1996). Although the fit isn't entirely natural, aspects of NOS can be categorized into four major areas based on general epistemology for the sake of comparison with nature of mathematics. These areas are: (a) *Certainty of knowledge*, (b) *Simplicity of knowledge*, (c) *Source of knowledge*, and (d) *Justification of knowledge* (Table 1). It cannot be overemphasized that these categories of general epistemology are not necessarily inclusive of all aspects of NOS, nor are we attempting to present a full and complete description of NOS in this report. Indeed, a problem with focusing on general epistemology, even for the sake of comparing disciplines, is the danger of losing site of the complexities of the disciplines. Nonetheless, with this stated caveat, certain aspects of NOS can be described using these four general areas of epistemology.

Certainty of scientific knowledge

Scientific knowledge is tentative. Scientific knowledge is partially the product of human inference and imagination and is contextually bound to the society and culture in which it is practiced. Levels of certainty can vary depending on the nature of the evidence and the internal consistency of the claims. That is not to say, however, that claims can ever be accepted as absolute truth. Science, by its very nature as a human endeavor and basis in the empirical world is subject to change. As such, no “proof” in any absolute sense is attainable.

Simplicity of scientific knowledge

Scientific models are created to offer explanations and predictions of observed phenomena. Models are not exact replicas of the phenomena, but rather models are representations that serve to explain, simplify, and predict. Further, scientific knowledge is complex in its internal consistency and interconnectedness.

Source of scientific knowledge

Scientific knowledge has its basis in empirical observation. Observations are made through the human senses or extensions of the senses. Meaning is given to observations, their relationships, and impact on each other through application of human creativity and currently accepted knowledge of science.

Justification of scientific knowledge

Scientific knowledge is justified through empirical observation and inference, and the development of investigations, explanations, and acceptance of knowledge relies on currently accepted theories, laws, and paradigms of the scientific community and society in which the science is practiced. As such, science is a complex social activity with a foundation in the empirical world.

Nature of Mathematics

The nature of mathematics (NOM) refers to the epistemology of mathematics, the features of mathematical knowledge and its development. Since the 1990’s, nature of mathematics has been included in mathematics education reform documents. While some may contend that the nature of mathematics is central to past and present mathematics education reform documents, NOM has never been a “standard” or “strand” (NCTM,1989, 2000). Nevertheless, a whole chapter in *Science for All Americans* (1990) was devoted to describing the nature of mathematics. Similar to aspects of NOS, several aspects of NOM are viewed as important and relevant to k-12 students. For the sake of comparison, the four general areas of epistemology are applied to describe NOM as follows (Table 1):

Certainty of mathematical knowledge

Mathematical knowledge is dynamic, and develops, like science, through human creativity and imagination. Newly created definitions as well as variations in the collections of axioms that make up mathematical systems serve as the main agents of change in mathematics. When imaginary numbers were defined and finally accepted by mathematicians, certain quadratic equations that were once considered “unsolvable” were deemed solvable in a new number system that included complex numbers. The development of non-Euclidean geometry was initiated by rejecting Euclid’s fifth postulate on parallelism. Thus, this slight variation in the foundation of the mathematical system describing spatial relationships yielded dramatic changes in the field of mathematics.

Simplicity of mathematical knowledge

Mathematical knowledge consists of a body of interrelated ideas, not isolated bits and pieces of information. From definitions and axioms of a mathematical system, new ideas are developed from existing ones through creativity and chains of logical thought. Logical thinking is also used when mathematical structures from existing systems are used in the development of other systems (such as complex number theory). While logical thought plays an important role in the consistency of mathematical knowledge, human creativity propels mathematicians to reason connections like those between co-existing systems (i.e., algebraic and spatial geometry systems).

Source of mathematical knowledge

Mathematical knowledge exists because humans have given meaning to their own thoughts and ideas. Mathematics is a human creation and thus, naturally subjective and arbitrary. Social and cultural influences play a role in the subjectivity and arbitrariness found in mathematics. An examination of earlier civilizations reveals how cultural influences guided the development of the Mayan numeration system, which was developed according to the civilization's religious and agricultural calendar. The writing tools chosen by the Babylonians (using reeds to mark wet clay) influenced their numeration system, and it is widely believed that the base 10 numeration system was developed from counting on fingers. Social influences, as well as cultural, add to the subjectivity of the development of mathematics itself. Current interest regarding on-line computer security has spurred further developments in number theory.

As a human creation, mathematical knowledge is also arbitrary. The properties of shapes, such as parallelism or congruency of opposite angles in some quadrilaterals, are features arbitrarily defined by mathematicians. Further, it is important to remember that arbitrarily define mathematical objects that consistent, that is, related to the other mathematical ideas and objects.

Justification of mathematical knowledge

Mathematical knowledge is justified through logical deductive proof based on generalizations and abstractions, not by observations of specific examples of mathematical phenomena. Observations help to convince others of conjectures, but alone do not suffice as the sole device for mathematical justification. In addition, while much of mathematics has developed from and can be applied to real-world situations, some mathematical knowledge has been created without reference to real world situations. For example, Buckminserfullerenes, or Bucky Balls, geodesic domes consisting of only hexagonal and pentagonal faces, were created prior to being discovered in some carbon molecule structures (Angel, 2000). While it is true that models of Bucky Balls can be constructed from materials in the physical world, mathematical proof serves as the only justification of this mathematical object.

Comparisons and Implications

Table 1 displays NOS and NOM in a side by side comparison. From this display, it becomes evident that there are many similarities as well as differences between the two disciplines. When looked at more holistically, however, a single factor emerges a fundamental difference. This is the reliance or not on an empirical foundation. Science demands basis in the real world for the source and justification of knowledge. In contrast, accepted mathematical knowledge can be a creation of the mind with no basis in the empirical world. This difference serves to define the natures of the disciplines and the processes by which scientific and mathematical knowledge is acquired. Inquiry and problem solving are ways in which questions can be addressed, but the disciplines of which they are a part differ by their very nature. Without consideration of these differences, the boundaries between mathematics and science may be blurred. In which case, formal understanding of either discipline would likely be unattainable.

Approaches to integration that advocate blurring or dissolution of disciplinary boundaries runs counter to the current emphasis of reform on inquiry and problem solving.

These higher levels of thinking skills require students to solve problems, make decisions, and/or draw conclusions based on data that have been collected, following either a self-designed or prescribed procedure. The key point, however, is what counts as data upon which decisions can be made (source and justification of knowledge). In scientific investigations and inquiries the critical data are that which is derived from the empirical world. In mathematics, proofs or problems do not require appeals to external empirical observations. The evidence comes from "inside" in terms of logic and logical relationships among knowledge claims. We certainly do not deny that there are multitudes of examples of mathematics principles in the empirical world. Certainly, mathematics has had applications to the world in which we live. However, these applications are not necessary (and often not even considered) for the establishment of the validity of mathematical knowledge claims.

Integrated curriculum approaches are typically based on problem/issues students are to solve. And, the logic of the integrated approach is that real world problems are not the property of the discipline as opposed to another. We heartily agree with this proposition. However, we are also well aware that certain conventions for what counts as evidence will need to be followed for students to arrive at their decisions and/or conclusions. This is not an artifact of classroom life. It is the same in the real world. Mathematics and science have different conventions for inquiry and problem solving and the conventions are mutually exclusive. Combining the two sets of conventions for evidence on an equal basis would result in a hybrid version of inquiry that is both confused and chaotic. It is not a question of which set of conventions is best for obtaining the "truth." Quite simply, different disciplines view the world and how one comes to know it differently. The conventions that describe these ways of knowing are what discriminate one discipline from another. The situation is not unique to science and mathematics. We could just as easily have included history, religion, art, philosophy, and music in this discussion.

We are not saying that any attempts to totally integrate science and mathematics in the same curriculum will result in irreversible arguments. At very basic levels, when there is no stress on data collection, little problem will arise. However, when the instruction approaches inquiry and problem solving, and data collection becomes important in reaching decisions, the incommensurate conventions of mathematics and science will cause problems.

Considering our original question of “Is mathematics a science,” we must state a resounding “No.” If the curriculum and/or instruction emphasize inquiry and problem solving, an equal balance between science and mathematics cannot and should not prevail. Mathematics can either be presented in a science context or mathematics will be used to help solve a scientific problem. Depending upon which conventions of evidence are given priority, one or the other discipline will necessarily dominate. There is no problem with attempts to organize instruction and curriculum in ways that combine numerous disciplines. We are not, however, in favor of approaches that attempt to ignore or dissolve disciplinary boundaries. Such approaches that work against the visions of reform that stress inquiry and problem solving and the value that each of our numerous disciplines brings to the solution of a problem or the development of new knowledge. To achieve deep understanding of the nature of the disciplines and concepts therein, we should be celebrating the diversity and integrity of the various disciplines as well as the benefits of differing perspectives.

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Table 1. Aspects of NOS and NOM as categorized by four general areas of epistemology.

Area of epistemology	NOS	NOM
Certainty	Tentative (subject to change); connections between observations and inferences/ evidence and conclusions influenced by human creativity and imagination and social/cultural values	Dynamic (subject to change)
Simplicity	Models serve to explain phenomena yet are not exact replicas; Internally consistent with currently accepted scientific knowledge	Interrelated and connected by logical chains of reasoning
Source	Empirical observations and interpretations which are theory-laden (subjective); human creativity and imagination, internally consistent with currently accepted scientific knowledge	Human creativity and imagination; arbitrary, yet consistent with other mathematical ideas
Justification	Empirical; internally consistent with currently accepted scientific knowledge	Logical deductive proof based on abstract, nonempirical mathematical objects