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AN EXPLORATION OF THE FOCUSING APPROACH FOR EXAMINING STUDENT DISCOURSE AMONG GEOMETRY STUDENTS

Graduate Project: A Thesis on

Adolescence Mathematics Education

by

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Abstract

The purpose of this study is to examine discourse among secondary geometry students while implementing the focusing approach. The study investigates interactions in the context of geometric reasoning and sense making. With the focusing pattern approach, the teacher asks questions and makes statements to place the control of the conversation back on the students. When this is done, the students articulate, reflect upon prior knowledge, and social network to create meaning and understanding of mathematical concepts. Through teacher facilitation and student collaborations, mathematical ideas are analyzed, synthesized, and evaluated with peers instead of looking for affirmation from the teacher. In this exploration, twelve students were placed in groups of three and given an activity. The researcher audio recorded the students while working on the activity. A protocol for the activity was developed and used by the researcher. The results show that main discourse occurring in the classroom was questioning, explaining, clarifying, justifying, reflecting, mathematical articulation, and listening. While the protocol was effective, there were times when the researcher began to provide guiding questions. The protocol was difficult to implement when students experienced difficulty answering the questions. Nevertheless, the students clarified and explained information and ideas to one another when a question was asked, focused on pertinent aspects of a solution when the researcher restated what a student said, and stayed attentive to the problem when comprehension questions were asked. In conclusion, discourse and the focusing approach can impact learning positively by increasing student engagement and conceptual thinking.

Key Words: Discourse, Geometry, Secondary Mathematics

Introduction

Communication in the classroom is very important for expanding students' understanding and comprehension of mathematics. According to the National Council of Teachers of Mathematics [NCTM] (2000), communication is "a fundamental element of mathematics learning" (p. 348). Through communication, ideas are presented and reflected upon, bringing reasoning and clarity to the learning condition (NCTM, 2000). Within communication, discourse encourages the construction of knowledge and understanding (Gilbert & Dabbagh, 2005). Gilbert and Dabbagh (2005) define discourse "as a process of collaboration and social negotiation where the goal is to share different viewpoints and ideas and collaborate on problem solving and knowledge building activities" (p. 6). Therefore, through interactions with peers and their instructor, students exchange ideas and make sound arguments by openly articulating mathematical concepts.

Wood (1998) illustrated two contrasting ways in which discourse can occur in the classroom: funneling and focusing. This exploration examined student discourse that transpired from the utilization of the focusing approach in a secondary geometry classroom. While student discourse is important for learning all mathematical topics, interactions were investigated in the context of geometric reasoning and sense making. In geometry, student discourse is very important for helping students' reasoning skills and to make sense of geometric concepts. In *Principles and Standards for School Mathematics* (NCTM, 2000), high school geometry is a "natural place for development of students' reasoning and justification skills culminating in work with proof in the secondary grades" (p.41).

Theoretical Framework

Communication is necessary in the process of learning and understanding in secondary mathematics classrooms. Communication and interaction in the mathematics classroom is widely supported in the mathematics education community (Griffin, Jitendra, & League, 2009). Constructivists find it pertinent for students to learn by actively creating and forming their own mathematical ideas and concepts through constructing mental schemas from connections made with prior knowledge and a transmission of information from a social network (Lau, Singh, & Hwa, 1999; Griffin, Jitendra, & League, 2009). Communication with a classmate or teacher encourages students to reflect on their reasoning, make connections, formulate and ask questions, divulge ideas, clarify understandings, and articulate clearly (NCTM, 2000).

Vygotsky discussed the importance of social interactions that are language mediated and how they are necessary in internalizing and evoking higher cognitive activity (Gnadinger, 2008). The utilization of spoken and written language, as well as listening, by the teacher and student, is imperative for the student to excel in knowledge construction of mathematical concepts and advance in mathematical reasoning skills and understanding.

The teacher plays a unique role in facilitating and guiding discourse. It is the teacher’s responsibility to help students feel secure and confident while communication occurs in the classroom. The teacher must also choose appropriate techniques that foster meaningful discourse and in turn amplifies the students’ learning process (Springer & Dick, 2006). Different communication patterns or techniques used among teachers and students may have stronger outcomes than others in building well-connected conceptual knowledge and understanding through articulation, social negotiation, reason, and reflection (Gilbert & Dabbagh, 2005; Griffin, Jitendra, & League, 2009; Wood, 1998). It has been found that students demonstrate higher level thinking skills when discussion practices are used within the classroom for instruction. When discourse occurs, students think, act, and remember information in a greater capacity. Discussion among students provides development in problem solving skills. Thus, alternative patterns to a traditional lecture have been the goal of many classrooms in over the past decade.

Wood (1998) described two main alternative communication patterns to traditional-oriented instruction occurring in mathematics classrooms: funneling and focusing. Funneling is portrayed when a teacher asks a series of questions that guide students though a problem. These questions are intended to evoke the teacher's own conceptions. In essence, the student is trying to produce the answer the teacher is looking for rather than engaging in active mathematical thinking.

Focusing is when a teacher asks a series of questions and make statements to assist students to reflect upon, reason with, make connections, and question themselves and others about mathematical ideas. When focusing occurs, learning is being conducted through a collaboration of interactions from teachers and students. The teacher asks students certain questions, has students explain and repeat their thoughts, listens to students' ideas, and rephrases fundamental statements. The teacher is letting the students evaluate and create mathematical ideas from other students instead of the students looking to the teacher for affirmation and answers to a problem. The "control of the conversation" (Wood, 1998, p.175) is also restored to the student articulating their thought instead of the teacher. After the teacher has facilitated the students' ideas, the students have created solutions to the problem. When most students understand the solutions, the teacher will then critique the solutions and may provide more insight on methods of solving the problem. By using focusing, mathematics is learned by articulation, reflection, and social negotiation creating meaning and understanding for the students (Wood, 1998; Gilbert, & Gabbagh, 2005).

Through discourse, students obtain the opportunity to improve on their reasoning skills and sense making. In *Principles and Standard for School Mathematics* (NCTM, 2000), geometry is “a natural place for development of students’ reasoning and justification skills culminating in work with proof in the secondary grades” (p.41). Geometry is related to many other mathematical topics, non-mathematical subjects, as well as to the world we live in today. Despite the importance of geometry, many more studies are needed in the exploration of students’ difficulties, of how to help students understand geometric concepts, make connections, and reason, and alternate approaches to learning (Battista, 2007).

During the 1960s, Pierre and Dina van Hiele developed a model to show how students understand and develop geometric reasoning and thinking. The van Hieles were two teachers who experienced difficulty teaching geometry and then developed classroom activities to improve students’ understanding of geometric concepts. Through observation, the van Hieles came to the conclusion that students generally progress through discrete, qualitatively different levels of geometric thinking (Fuys, Geddes, & Tishler, 1988). The van Hiele model consists of five levels, ranked by order of difficulty, and students must accomplish and exceed each level to proceed to the next level. The levels are as follows:

Visualization (Level 0): Students are able to recognize geometric figures by observations of appearance. Students, at this level, will determine names and shapes by the visual whole and not consider individual properties.

Descriptive/analysis (Level 1): Students are able to examine and recognize geometric concepts. They begin to differentiate and experiment with different properties, characteristics, and relationships of geometric structures.

Ordering (Level 2): Students develop and are able to order and interrelate logical relationships between shapes and geometric figures. They are able to infer and make implicit and informal arguments pertaining to previously discovered geometric concepts and properties.

Deduction (Level 3): Students develop an understanding of theorems in an axiomatic system. The students are able to define, explain, and utilize theories to conduct formal proofs deductively.

Rigor (Level 4): Students are able to understand theorems in different axiomatic systems. At this level, students should be able to analysis, compare, and utilize information about deductive systems.

The decisions a geometry teacher makes when teaching in the classroom is essential to a student's understanding of geometric concepts. Using the van Hiele model, a teacher notices the level of each student and then modifies her instruction and classroom activities to match those levels. A teacher must also continue to listen and pay attention to students' levels because other researchers have found that students tend to flow from one level to another instead of jumping in one complete transition (Battista, 2007). Thus, teachers may see a student working in two different levels. To help students transition from one level to another, the teacher elicits focusing questions for recognition of salient aspects of a solution (Sharp & Zachary, 2004).

The Purpose of the Study

The purpose of the study is to explore student discourse through the utilization of the focusing pattern in a secondary geometry classroom. The study will investigate interactions in the context of geometric reasoning and sense making through an activity.

Method

Participants

The study group consisted of 12 students from a secondary classroom. The classroom enrollment was 16 students; 4 students were non-participants. Seven female and five male students participated. Twelve participants were: 7 sophomores, 4 juniors, and 1freshman. The participants are typically taught by direct instruction. All participants in the study group were native English speakers and Caucasian. The study group originated from a middle class public school district in suburban rural New York.

Design of the Study

The study was completed in one class period lasting approximately 40 minutes. The participants were placed in four groups. Each group consisted of three participants. The participants were given three sheets of paper and one pair of scissors. One sheet of paper was an activity (see Appendix A) for the students to complete, the second sheet of paper was plain and meant to accompany the activity, and the third sheet of paper was also plain and given to the students as an extra piece of paper in the event more room was needed for written work. The activity sheet provided a series of questions in order to accommodate all participants. The activity included questions ranging from 0 to 3 van

Hiele levels. Since it is likely that the participants had a range of developmental reasoning skills, it was important to pick an activity in which students at any level would be able to participate in the discussions.

I conducted the study with the participants while the classroom teacher worked with the non-participants. I visited the classroom nine times in a three-week period prior to the study occurring. Visiting the study group allowed me to develop some rapport with the participants. I developed a focusing pattern protocol prior to working with the participants (see Appendix B). Each group was audio-recorded. Following the group work, I transcribed the audio-recordings. The transcriptions were used in examining the participants' discourse.

Data Analysis

The analysis of the data occurred in two stages. The first stage involved listening to the audio recordings of the four participating groups in order to describe student discourse, transcribing and reading the data. First, I transcribed all of the data from each of the four groups. Second, I looked for similarities in the way students were talking and looked for ways discourse occurred between the students. The categories were: questioning, explaining, clarifying, reflecting, justifying, reconstructing ideas, articulating mathematically, listening, and gathering additional facts. Third, the classroom teacher and I listened to the audiotapes together so that I was able to further clarify students' discussions. Fourth, the classroom teacher validated the categories by using a sheet created specifically for this data while reading the transcripts. Fifth, the categories were adjusted until both the classroom teacher and I agreed 100%. The classroom teacher and I added types of discourse while listening to the audio recordings and read the transcriptions. Instructing, re-stating, and logical reasoning were included when the audio recordings and transcripts were revisited.

The second stage of analysis was to evaluate the protocol made from the focusing pattern approach to learning (see Appendix B). I extensively read the transcripts from the four groups participating to answer three questions:

1) What pieces of the protocol were used?
2) What questions or comments were made instantaneously or on the spot?
3) How effective was the protocol?

Results

The results are provided in two sections. First, I will describe the student discourse that occurred during the activity. Second, I will describe the effectiveness of the protocol that was developed for this activity to utilize the focusing approach.

Student Discourse

I found that questions, explanations, clarifications, justifications, reflections, mathematical articulations, and listening occurred often throughout the activity. Re-constructing ideas occurred, but as a combination of reflection and clarification. Other discourse categories that occurred but were not as abundant were:

1. Gathering additional facts;
2. Instruction;
3. Re-stating information;
4. Logical reasoning.

Questioning

Students used questions to interact with one another in the retrieval of information from fellow students and oneself. Students asked questions for clarification on directions, analysis of the given problem, and verification. For example, one student, who used questioning to analyze and verify, stated, “So, for two, we can say that it just depends on the shape of triangle you’re using?” Another student used questioning to clarify by stating, “Do we just draw a triangle?”

Explaining

Students also explained when providing reasons for an answer or when helping a student complete a task. For example, one student offered that putting the large triangle upside down can make another triangle as an explanation for his reasoning. Another student disagreed and stated, “You’re rotating it.” In another example, a student explained, when helping a student with directions, by stating to another student, “Then fold it in half again, like this.”

Clarifying

Clarification occurs when ideas are made clear, distinct, and apparent. Clarifying occurred mostly when a question was answered, explanations were vague, and ideas needed better wording. For example, one student clarified how to put the triangles together, and said, “You put them all like together, no you have to put all four of them together.” The student began to explain what the other student should do, but then clarified his explanation.

Justifying

Justification was observed in the discourse when students verified their thoughts or reasoning. For example, one student, who was trying to figure out if a larger triangle could be made from the four small congruent triangles said, “No, that wasn’t it. I don’t think you can. Can you? What if I made like a, one of my sides is like not straight. That’s right, you can. I did have it, my sides just aren’t straight.”

Reflection

The participants used reflection when they thought seriously about previous information discussed or known. In most cases, the participants cogitated in hopeful anticipation of answering a question. For example, one student said, “Okay, also the angles, oh, the angles stay the same, stay the same, the angles have the same measure, is that what it’s called, the angles.” This student was reflecting on what had previously been said and seemed to realize that the sum of the angle measurement of the small triangle and the sum of the angle measurement of the large triangle both add up to 180°. After the topic was changed for a brief moment, the same student said, “Also, the angle measures stay the same even in the triangle that’s bigger, four number four, yeah that is true, it’s a postulate I think or a theorem or something.”

Articulating Mathematically

Students used mathematical vocabulary and concepts in their discussions. For instance, when students were trying to figure out if a small triangle had anything in common with the large triangle, one student said, “It’s that same triangle. It’s the same congruent triangle, just dilated.” Other mathematical vocabulary words and concepts used to express a thought clearly and coherently were; isosceles triangle, base angles, vertex, right triangle, equilateral triangle, congruent, rectangle, square, angles add up to 180°, remote angles, quadrilateral, parallel lines, and supplementary.

At times, I noted that students struggled with vocabulary or used non-mathematical vocabulary. For instance, one student said, "I've got *stuff* like three sides, three angles." Here the student could have used the word 'properties' rather than 'stuff'. Another student said, while cutting out the triangle, "I'm gonna make a *tiny* one, mine are *tiny*, mine are *babies*, they look like the smaller triangles."

Listening

Listening occurred often, in all four groups, throughout the class period. The students were able to ask questions and receive a response, get help by explanation, receive clarification, discuss opinions or ideas, and make arguments because they listened to one another. The following dialog demonstrates that students seemed to be listening to each other during the interaction:

Student 1: Now have four congruent triangles, can you put the four triangles together to make a large triangle? Oh, I get it, really."

Student 2: So wait, you cut it out like this?

Student 3: Yes, just out like a triangle, like cut yours out.

Student 1: Yeah, you can look how I made mine.

Protocol Effectiveness

The protocol was designed with the focusing pattern in mind. The protocol consisted of questions and statements I developed to increase student interaction and to help students develop their own ideas. While comparing the protocol with the actual questions and statements offered during the activity, I used several of the questions from the protocol (see Table 1). The main pieces of the protocol used were:

1. Asking students to explain or repeat their thoughts;
2. Seeing if students understood what was just said;
3. Getting students to explain what they know about triangles;
4. Asking students how to prove a proof;
5. Calling for an agreement of what was said.

I did not, however, use all of the protocol. I did not use protocol that consisted of what students thought an answer should be, how they thought something should be solved, pertaining to directions, comparing work with other students, why a method works, and if anyone had the same answer. Also, all questions and statements were not asked verbatim from the protocol or directly correspond to the activity question they were placed under. For instance, I said, “Maybe you can help explain this to some of your fellow students.” In this situation ‘this’ means the configuration of the large triangle. The protocol said, “_____, will you explain to the group and me what you did to make a large triangle” or “Ask a student to explain their train of thought to other students.” An example of a question I asked that did not directly correspond to the question in the protocol is similar to, “What do you need to find out?” In the protocol, the previous question asked is placed under question two, but I asked it when students were trying to figure out the answer and discussing Question 3.

Table 1: Protocol Implemented into the Classroom

Protocol Used	Protocol Not Used
1. _____, Will you explain to the group and me what you did to make a large triangle?	1. How do you think this should be solved?
2. Restate what one student has said when discussing his/ her/ thought with other students.	2. What have the directions asked you to do?
3. What do you need to find out?	3. _____, What have you don't to find the answer?
4. Do you understand what was just said/ explained?	4. How do you think this should be solved?
5. What information/facts do you have?	5. Why do you think there are_____ number of ways to create a large triangle?
6. What do you know about triangles?	6. What do you think the answer or the result will be?
7. How have you proved proofs in the past?	7. Have you compared your work with anyone else's, what did they do?
8. Will you explain your reasoning again to other students and me?	8. What strategy are you going to use?
9. Why is that true?	9. Can you explain why your method works to the groups and me?
10. What do you think about what _____ said?	10. Can you describe your methods you have used to prove the new triangle is actually a triangle to the group and me?
11. Ask a student explaining information to other students a specific part of their reasoning?	11. Does anyone have the same answer, but a different way to explain it?
12. Can you explain what you have done so far? What else is there to do?	12. Have you compared your thoughts with anyone else, what did they try?
13. Do you agree? Why or why not?	13. Ask a student if they are finding the responses to be the same or different from the first weird triangle they worked with.
14. What do you know about the larger and smaller triangles?	
15. Ask a student to explain their train of thought to other students.	
16. Think about the concepts you have learned about triangles. What have you learned?	
17. Repeat what a student has said.	
18. Will you explain your reasoning again to other students and me?	

As I was working with the students, the questions and statements similar to the protocol were not the only questions and statements used. Questions or comments were asked and made instantaneously or on the spot. Most of the questions asked were focusing questions or questions that turned the conversation back to the students. Some questions were,

1. What are some of the properties of triangles?
2. Okay, can the configuration be different?
3. Why don't you write down what you know about a triangle?
4. Do you think your configuration will look similar to your groups?

I relinquished the control of the conversation back to the students often, and made encouraging comments and gave feedback on occasion. In addition to re-stating or re-phrasing the students' comments, I made statements showing praise and encouragement of a student's work ethic and ideas.

I felt the protocol was effective. When I placed the control of the conversation back on the students, the students had to think for themselves and with others. Thus, the students did not rely on me. The students were able to question, explain, clarify, justify, articulate, listen to, and reflect upon mathematical concepts. The following illustrates an effective portion of the protocol, which occurred when a group had previously discussed question number four and thought a student's answer was weird.

Betty: What is, what did you say?

Student 1: I said like that the bigger ones look like the small ones.

Betty: Do you guys find that to be true?

Student 2: I don't know what she is saying.

Betty: Would you, the bigger one looks like the small one. Would you mind explaining that?

By rephrasing and asking for an explanation of what the first student said, I was able to turn the conversation back to the student for clarification and showed that the first student's statement was important. Therefore, the students were able to strategize and focus on the problem for its solution. I was also able to bring the other students into the conversation by having the first student explain her way of thinking to them, as suggested by Wood (1998).

Student 1: The small one, the small one, the small one, put them together it looks just like it just a bigger size.

Student 2: Oh, okay.

Betty: Do you understand?
(Talking to the group)

At this point, all of the students understood what their group member was saying and agreed with her. By asking the students if they have comprehended what was said, I attempted to have the group focus on the content provided by the first student, which increased their attention to the problem.

Also, while studying the transcripts I noticed that when the students became frustrated with uncertainty of how to solve a problem they were more likely to veer off task. When the protocol was used, less frivolous discussions and increased mathematical interaction pertaining to the activity occurred. Thus, student engagement increased productivity. When I used the protocol, students seemed to be on task and focused, which provided and environment for meaningful discourse. For example, one student caused a major distraction for her group. Minimal work was accomplished on the activity when I was not there to focus the students using the protocol.

Betty: What’s going on over here?
(I realized that the group was not discussing the activity)

Okay, let’s talk about the project going on, okay?
(I looked at one of the students’ answer written down to question number two.)
So, you have one way?

Student 1: I had another way here.
(Unfortunately, I did not hear this student.)

Betty: What did you say before?

Student 2: I had one way with one kind of triangle, but another way with a different, with a right kind of triangle.

Betty: You have a different way with a right kind of triangle. Does everyone understand what he is saying?

Student 2: Yeah.
(The other student shook their head no).

Betty: Will you please explain that to her?

The previous time I had come to this group, they had been working on the same question, and found out that four congruent right triangles could be configured two ways to create a large figure looking like a triangle. One of the students seemed to be having a hard time working with the triangles and answering the questions. This student seemed to take the other student’s attention away from the activity. I observed that I needed to focus the group on the activity and to move forward. In this process, I guided and redirected the group for further clarification and reflection.

The protocol seemed limited when the students were not able to answer a question. When the students could not figure something out after much debate, I observed that the students were becoming restless and I started to guide and funnel the students. Students seemed most restless while discussing Question 3. Many of the groups figured out that, if a figure has three sides and the sum of the interior angles was 180°, then it is a triangle. Students experienced difficulty when figuring out why the three sides were straight and how the large figure had angles that when added together equal 180°. In one scenario, I guided the students in helping them write a proof and use what is given in the activity to help them prove that the larger figure is in fact a triangle.

Betty: All right, so, how do you prove a proof?

Student 1: Awe, I'm not the one to do this thing. How do you prove a proof?

Betty: Yes.

Student 1: It has three sides, that was his answer, um.

Betty: How do you already set it up?

Student 1: Statements and reasons.

Betty: Very nice, statements and reasons, does everyone agree with that?

Student 1: Yup, I really hope we don't have to do a proof.

Betty: What is the first statement and reason usually?

Student 1: Whatever's given.

Betty: Okay, let's look at what's given, what's given?

Student 2: A new triangle is created.

Student 1: What?

Student 2: A new or larger triangle is created, a new triangle, so a new triangle is created.

Betty: No, it says prove that a new triangle is created, this is just a figure, it might not be a triangle, right? So, but what do you know, what's given?

Student 2: Um.

Student 1: Four congruent triangles.

Betty: Exactly, these are triangles, right?

In this situation, focusing was still occurring because the thinking was returned to the students. The students eventually found the properties of the small triangle and use those properties to show that the larger figure is in fact a triangle. Unfortunately, towards the end of the dialog, the focusing pattern ceased because I gave the students the answer I inquired from them. In another situation pertaining to Question 3, funneling occurred. In this situation, I point to a side and three vertices and start to discuss Question 3.

Betty: How do you know that those are sides? How do you know that all of these vertices put together create a side?

Student 1: They are all congruent, and they are all together.

Student 2: Well no, we have to think of some set up.

Student 1: What kind of set up should we think about?

Betty: How do you know that this is a side, a straight side?

Student 1: Cause these are congruent and when they're put together, when certain sides are put together, these sides can be isosceles so they can be the same side.

Betty: What makes something straight?

In this scenario, I am leading the students to the answer I seek. I am telling the students that when the measurements of the three vertices are added together they create a straight line. I am trying to have the students realize a side of a triangle consists of a 180° angle or straight line.

Discussion

The purpose of the study has been to explore student discourse through the utilization of the focusing pattern in a secondary geometry classroom. This study shows that meaningful discourse is demonstrated when the students collaborated and created arguments while discussing and interacting with peers and their teacher. Meaningful discourse is the exploitation of critical thinking skills, including analysis, synthesis, and evaluation (Gilbert & Dabbagh, 2005).

During the activity, students were able to get help from each other through questioning. Students made statements for clarification, explained interpretations, and offered input on ideas. Students articulated mathematically, justified ideas, and reflected on previous knowledge and contributions made from others. Lastly, students respectfully listened to each others' points of views.

When I used the protocol, the students seemed to be engaged with the activity. The protocol served as a technique not only to keep the students engaged, but also to help students attend to the task, to pay attention to specify important aspects by rephrasing mathematical ideas, to ask clarify and justify questions, to reflect on what others said, and to strategize and think out loud. The protocol also seemed to keep the students on task by turning the power of the conversation back on the students and by directing students to the pertinent facets of the solution through rephrasing. These interactions seemed to helped students reason and make sense mathematically.

However, it was difficult to not answer the students' questions directly or guide them differently. For instance, I continually gave the students encouragement for the ideas they discussed, which was not part of the protocol. It would also be very difficult to solely use the focusing pattern for teaching a curriculum. The class I worked with learned from a mostly traditional classroom setting. The students looked to me, as an expert, in anticipation of me teaching the student or telling the students answers and if they are on the right path to a solution.

Using only the focusing pattern approach to teaching seemed to be difficult at times during the activity. When the students were frustrated or not able to come up with a solution, I felt that they needed more guidance. This is when I tended to funnel the students. I did not intentionally stray from the protocol. I believe funneling the students occurred because I was feeling pressure to have the students' produce a valid proof as evidence of their learning. Also, many of the groups were very close to solving Question 3 but were not grasping a few key concepts. As a result, I helped them with missing concepts without using the focusing pattern.

Other occurrences for not using the protocol flawlessly are considered. Being a novice teacher and the students' inexperience of participating in the focusing pattern could also be factors. Looking at the transcripts, I felt that if I had more experience in teaching and using the focusing pattern I could have generated different questions and used the focusing pattern entirely. In spite of my experience, I observed that the focusing pattern approach did make a difference and students accomplished more than when just interacting within the groups.

Using a focusing pattern can take time to implement into a curriculum and can be difficult to use often. It takes time for students and their teachers to become acquainted with the approach. Other approaches seem inevitable because of time constraints on the curriculum and the necessity of having some prior knowledge on the mathematical concepts in order to solve some problems.

Constructivists believe that having the students learn through one another is a productive way of learning. Social learning structures, when compared to non-interactive classes, can be intimidating to some students. Wood (1998) discussed how some students can become at risk when socially interacting mathematically. This could be for a variety of reasons. One reason would be that a student does not feel confident in their answer. It was observed that some students appeared to enjoy or not mind the group engagement activity while discussing their ideas while others were found to be less vocal.

Nevertheless, the teacher is responsible for creating a productive learning environment. For the purpose of having students work together and using the focusing pattern, the teacher must create a classroom environment in which the students feel comfortable. This includes having the students show respect to one another and their point of view and the teacher showing respect to the students. The students I worked with seemed very comfortable with one another and definitely showed respect toward most other students and myself. For instance, when I was working with a group, the group behaved appropriately while participating with myself and other students.

Although student discourse and the focusing approach seem to be worthy of implementing into a classroom, further research is needed. Other variables could have impacted the results.

These include:

1. When using the focusing pattern, what differences occur among various group structures?
2. If the students are placed in a group, what personalities should have been placed together to create higher discourse for everyone?
3. Does the level mental capability from the van Heile theory impact discourse?
4. To examine student discourse, should a non-kinesthetic activity be presented?
5. How do students with various learning style interact when a focusing pattern is used?

To address some of these questions, background and personal information must be interpreted and taken into consideration about the students. Some of the information would be the students:

1. Intellectual capacity;
2. Expertise or knowledge base in geometry and mathematics;
3. Learning style;
4. Ability to self-regulate learning;
5. Ability to work well with others.

In addition, students' understanding of mathematical concepts both before and after the study took place would help research pertaining to if student discourse influences student's knowledge construction. Lastly, there is not an extensive amount of research on student discourse or the focusing pattern in secondary schools. More research is pertinent in both categories to increase student learning and pedagogy.

Conclusion

Throughout this study, communication, collaboration, and teacher interaction are important to student understanding of mathematical concepts. When I applied the focusing pattern, students' engagement and concentration levels seemed to be high. The exploration demonstrated that students applied prior knowledge to the mathematical concepts through social interaction to solve the problem. In this process, students analyzed, synthesized, and evaluated the information given, clarified with peers, justified answers, and reflected upon individual and group responses in finding the correct answer.

It was observed that students desired to produce correct answers. Some students became frustrated when confused about solving the problem. Students often looked to me for guidance and support to complete the task. The protocol served as a technique and tool to focus the students and their conversation on the assignment. The findings from this exploration suggest that further research be conducted regarding discourse and the focusing approach in geometry at the secondary level. Secondary mathematics teachers should be encouraged to engage students in discourse through the focusing approach. Thus, providing students opportunities for understanding through communication, reasoning, and sense making should be a valuable part of instruction.

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Appendix A

Weird Triangles Exploration

Directions:

Take a blank piece of paper. Fold it in half and then fold it in half again.

Draw a "weird" triangle — that is, any triangle — and then cut it out. You should now have four congruent triangles.

For each of the following questions, explain your reasoning.

1. Can you put the four triangles together to make a large triangle?

2. If question one can be completed, is there only one way, or are there different ways to create a large triangle?

3. If a larger or new triangle is created, prove that the new triangle is actually a triangle, as opposed to a figure that is almost a triangle.

4. If a larger triangle is created, does the new large triangle have anything in common with the original triangle? Please explain your answer.

5. Repeat this activity with other weird triangles until you are satisfied with your responses to the four questions.

http://college.cengage.com/mathematics/bassarear/elementary_school/2e/students/downloads/exploration8-8.pdf

Weird Triangle Exploration Possible Answers

1. Four congruent triangles can be arranged together to form a large triangle. Please refer to the image below.

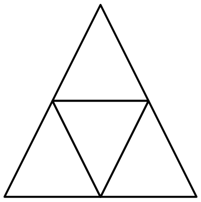

Figure 1: Arrangement for an isosceles triangle.

2. There are different ways in forming a large triangle from four congruent small triangles. For every triangle, excluding a right triangle, there is only one way to arrange the four smaller congruent triangles when making a large triangle. This is because the vertex angles have to add up to 180° and the three vertices joined to make straight lines or sides of the large triangle have to add up to 180°. In an isosceles and equilateral triangle some of the smaller triangles can be flipped or rotated because some or all angles are congruent, but the formation of the small congruent triangles stays the same.

For right triangles, the four smaller congruent triangles can be arranged in two ways to make a large triangle.

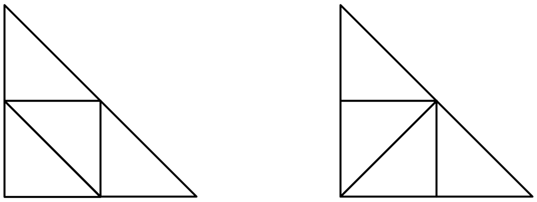

Figure 2: Arrangements for a right isosceles triangle.

3. I will prove that the larger figure is a triangle by using Euclidean Geometry and from a high school geometry student perspective.

A solution to prove that the large figure is actually a triangle will be shown by a synthetic proof. Synthetic proofs are commonly used in Geometry classrooms today.

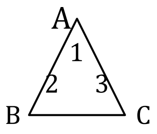

Figure 3

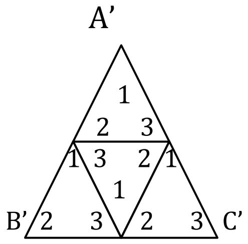

Figure 4

Proof

Statement	Reason
1. In Figure 3, ABC is a triangle	1. Given
2. The angles of ΔABC added together equal 180° m<1 + m<2 + m<3 = 180° m<ΔA′B′C′ = 180°	2. The sum of the measures of the angles of a triangle is 180°.
3. A′C′, B′C′, and A′B′ are straight lines	3. Straight lines are lines that have angle measurements equal to 180° when added together.
4. In Figure 4, A′B′C′ is a triangle	4. By 2 and 3 above

4. The new triangle does have something in common with the original triangle. The two triangles are similar triangles. If two polygons have corresponding angles that are congruent and corresponding sides that are in proportion, then the polygons are similar

Appendix B

The majority of the questions below pertain to asking students to explain or restate what was just said, if they understand what is being explained, if thoughts have been discussed with their peers, information already known, asking a student if they believe another student's statement to be true and why, asking students what they think the answer may be, and strategies of how to answer a question. The statements said to the students were repeating what a student has said. Repeating or rephrasing a student can turn the attention of the students on an important part of the solution (Wood, 1998).

Protocol for Implementing the Focusing Pattern

I will be asking students questions through the focusing pattern to presumably encourage student discourse and for students to stay on task. Execution of the focusing pattern will occur while the students are completing an activity: Weird Triangles Exploration. I will use the focusing pattern to best fit a scenario.

Questions from Weird Triangle Exploration and Examples of Potential Focusing Pattern Implementation:

1. Can you put these triangles together to make a large triangle?
    a. How do you think this should be solved?
    b. What have the directions asked you to do?
    c. ____, what have you done to find an answer?
    d. ____, will you explain to the group and me what you did to make a large triangle?
    e. Restate what one student has said when discussing his/her thoughts with other students.

2. If question one can be completed, is there only one way, or are there different ways to create a large triangle?
    a. How do you think this should be solved?
    b. ____, will you explain to the group and me what you did to make a large triangle?
    c. Why do you think there are _ numbers of ways to create a large triangle?
    d. Restate what one student has said when discussing his/her thoughts with other students.
    e. What do you need to find out?
    f. What do you think the answer or the result will be?
    g. What strategy are you doing to use?
    h. Have you compared your work with anyone else's, what did they do?
    i. Do you understand what was just said/explained?

3. If a larger or new triangle is created, how could you prove that the new triangle is actually a triangle, as opposed to a figure that is almost a triangle?
    a. What information/facts do you have?
    b. What information do you need to find out?
    c. What do you know about triangles?
    d. What strategies are you going to use?
    e. What do you think the answer or result will be?
    f. ____, will you explain to the group and me what you did to prove that the new triangle is actually a triangle?
    g. Have you compared your work with anyone else's, what did they try?
    h. How have you proved proofs in the past?
    i. Restate what one student has said when discussing his/her thoughts with other students.
    j. Will you explain your reasoning again to other students and me?
    k. Ask a student explaining information to other students a specific part of their reasoning.
    l. Can you explain why your method works to the group and me?
    m. Can you describe your method you have used to prove the new triangle is actually a triangle to the group and me?
    n. Can you explain what you have done so far? What else is there to do?
    o. Why is that true?
    p. What do you think about what ___ said?
    q. Do you agree? Why or why not?
    r. Do you understand what was just said or explained?

4. If a larger triangle is created, does the new large triangle have anything in common with the original triangle? Please explain your answer.
    a. What do you know about the larger and smaller triangle?
    b. Think about the concepts you have learned about triangles. What have you learned?
    c. Ask a student to explain their train of thought to other students.
    d. Repeat what a student has said.
    e. Have you compared your thoughts with anyone else, what did they try?
    f. Do you understand what was just explained?
    g. Do you agree? Why or why not?
    h. Will you explain your reasoning with other students and me?
    i. Will you explain your reasoning again to other students and me?
    j. Ask a student explaining information to other students a specific part of their reasoning.
    k. Does anyone have the same answer, but a different way to explain it?

5. Repeat this activity with other weird triangles until you are satisfied with your responses to the four questions.
    a. Have you conversed your thoughts with your group? What are they thinking?
    b. Do you agree with the other student? Why or why not?
    c. Ask a student if they are finding the responses to be the same or different from the first weird triangle they worked with.
    d. Will you explain your findings to the group and me?
    e. Repeat what the student has said.
    f. Ask a student if they understand what was explained to them from another student.
    g. What do you think the answer or result will be?
    h. What do you think about what ____ has just said?