Understanding and Using Availability

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Webinar Take-Aways

- Understanding Availability from a practical standpoint
- Calculating different Availability ratings
- Practical and Economic ways of enhancing Availability

Summary

Availability is a performance measure concerned with assessing a maintained system or device, with respect to its ability to be used when needed. We overview how it is measured under its three different definitions, and via several methods (theoretical/practical), using both statistical and Markov approaches. We overview the cases where redundancy is used and where degradation is allowed. Finally, we discuss ways of improving Availability and provide numerical examples.

When we can use Availability

- When system/device can fail and be repaired
  - During “maintenance”, system is “down”
  - After “maintenance”, system is again “up”
- Formal Definition: “a measure of the degree to which an item is in an operable state at any time.” (Reliability Toolkit, RIAC)

System Availability

- Is a probabilistic concept, based on:
- Two Random Variables: X and Y
  - X: System or device time between failures
  - Y: Maintenance or repair time
- Long run averages E(*) of X and Y are:
  - E(X): Mean time Between Failures (MTBF)
  - E(Y): Expected Maintenance Time (MTTR)

AVAILABILITY formula: A_o

$$A_o = P(System.Up) = \frac{Fav.Cases}{Tot.Cases}$$

$$= \frac{Up.Time}{Cycle.Time} = \frac{E(X)}{E(X) + E(Y)}$$

$$= \frac{MTBF}{MTBF + MDT}$$

Availability by Mission Type

- **Blanchard** (Ref. 2) states that **availability** may be expressed differently, depending on the system and its mission. There are three types of Availability:
  - Inherent
  - Achieved
  - Operational

Inherent Availability: A_i

* Probability that a system, when used under its stated conditions, will operate satisfactorily at any point in time.

* A_i <u>excludes:</u> preventive maintenance, logistics and administrative delays, etc.

$$A_i = \frac{MTBF}{MTBF + MTTR}$$

Achieved Availability: A_a

* Probability that a system, when used under its stated conditions, will operate satisfactorily at any point in time, when called upon.
* A_a <u>includes</u> other activities, such as preventive maintenance, logistics, etc.

Operational Availability: A_o

* Probability that a system, when used under its stated conditions, will operate satisfactorily when called upon.
* A_o includes <u>all factors</u> that contribute to system downtime (now called Mean Down Time, MDT), for <u>all</u> reasons (maintenance actions and delays, access, diagnostics, active repair, supply delays, etc.).

Illustrative Numerical Example:

Event	SubEvent	Time	Inherent	Achieved	Operational
Up	Running	50	50	50	50
Down	Wait-D	10			10
Down	Diagnose	5	5	5	5
Down	Wait-S	3			3
Down	Wait-Adm	2			2
Down	Install	8	8	8	8
Down	Wait-Adm	3			3
Up	Running	45	45	45	45
Down	Preventive	7		7	7
Up	Running	52	52	52	52

UpTime	147	147	147	147
Maintenance		13	20	38
Availability		**0.9188**	**0.8802**	**0.7946**

Formal Definition of Availability

Hoyland et al (Ref. 1): *availability at time* t, denoted A(t), is *the probability that the system is functioning (up and running) at time t*.

X(t): the *state* of a system at time "t"

* "up" (and running): [X(t) = 1],
* "down" (and failed): [X(t) = 0]

A(t) can then be written:

$$A(t) = P\{X(t) = 1\}; \quad \text{for } t > 0$$

Availability as a R. V.

$$A = \frac{X}{X+Y}; X, Y > 0$$

- The problem of obtaining the "density function" of A
  - Is resolved via variable transformation of the joint distribution
- Based on the two Random Variables: X and Y
  - time to failure X; and time to repair Y
- Expected Value and Variance of r.v. Availability (A)
- L_{10} (10th Percentile of A) = P{A < 0.1} = 0.1
  - First and Third Quartiles of r.v. Availability, etc.
- Theoretical results, are approximated by Monte Carlo

Theoretical Transformation Procedure

$\mathrm{X(i)} = \mathrm{Exponential}$ *transformed To:*

$y_1 = \frac{x_1}{x_1+x_2}, y_2 = ax_1 + bx_2,\ a\ and\ b\ constants$

Inverse Functions: $x_1 = x_1 y_2 + x_2 y_1$

and $y_2 = ax_1 + bx_2$

Joint distribution of y_1, y_2:

$$\mathrm{g}(y_1, y_2) = |J| f[\omega_1(y_1, y_2), \omega_2(y_1, y_2)]$$

Resulting in:

The Jacarobian J is :

$$\begin{vmatrix} \dfrac{\partial \omega_1(y_1 y_2)}{\partial y_1} & \dfrac{\partial \omega_1(y_1 y_2)}{\partial y_2} \\ \dfrac{\partial \omega_2(y_1 y_2)}{\partial y_1} & \dfrac{\partial \omega_2(y_1 y_2)}{\partial y_2} \end{vmatrix}$$

The Joint Distribution then becomes:

$$\mathrm{g}(y_1, y_2) = \mathrm{e}^{-(x_1+x_2)}|J|$$

NOT AN EASY WORK; WE SIMULATE!

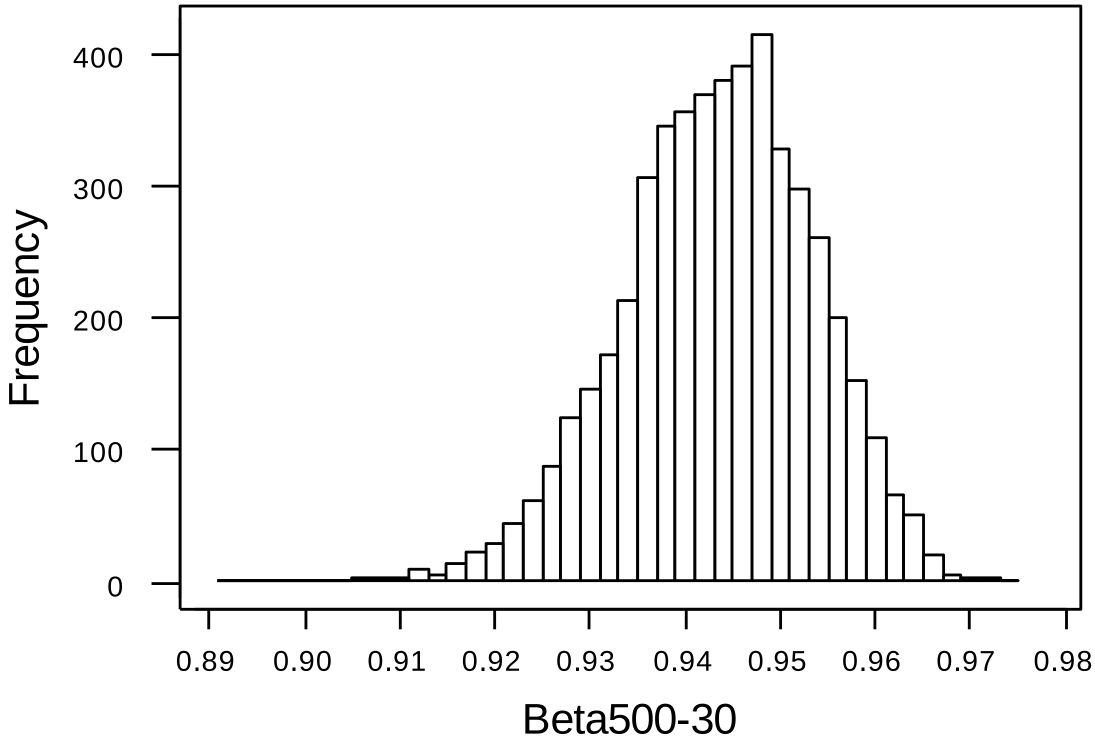
Monte Carlo Simulation

- Generate n = 5000 random Exponential failure and repair times: X_i and Y_i
- Obtain the corresponding Availabilities: $A_i = X_i / (X_i + Y_i); 1 \leq i \leq 5000$
- Sort them, and calculate all the n = 5000, numerical A_i results, $1 \leq i \leq 5000$
- Obtain the desired parameters from them.

Numerical Example

- Use Beta distribution for expediency
  - If X and Y are Exponential, then:
  - Ratio yielding A_i is distributed Beta(μ_1; μ_2)
- Time to failure (X) mean: $\mu_1 = 500$ hours
- Time to repair (Y) mean: $\mu_2 = 30$ hours
- Generate n = 5000 random Beta values
  - with the above parameters: μ_1 and μ_2
- Obtain the MonteCarlo Availabilities: A_i

Histogram of the Example:



Estimated Parameters of Example

MC Results for Beta(500,30) Example:
Average Availability = 0.9435
Variance of Availability = 9.92×10^{-5}
Life L_{10} = 0.9305
Quartiles: 0.9370 and 0.9505
P(A) > 0.9505 $\approxeq$ 0.269

$$P\{A > 0.95\} = 1 - P\{A \leq 0.95\} \approx 1 - \frac{3673}{5000} = 1 - 0.7346 = 0.2654$$

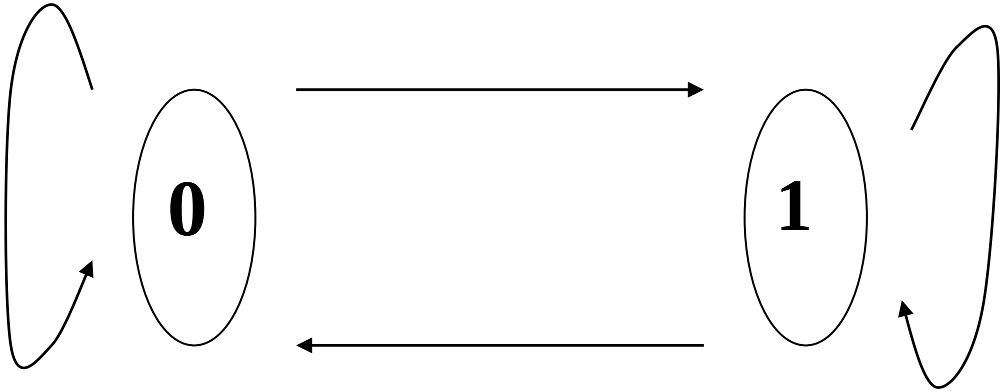
Markov Model Approach

- Two-state Markov Chain (Refs. 4, 5, 6, 7)
- Monitor *status* of system at time T: X(T)
- Denote State 0 (Down), and State 1 (Up)
- Let X(T) = 0: system S is down at time T
- Define the probability "q" that system S is Up at time T (or "p", that S was Down at T) given that it was Down (or Up) at time T-1?

Markov Representation of S:

$$p_{01} = P\{X(T) = 1 | X(T-1) = 0\} = q$$

$$p_{10} = P\{X(T) = 0 | X(T-1) = 1\} = p$$



Numerical Example:

- System S is in state Up; then moves to state Down in one step, with Prob. p_{10}= p = 0.002
  - A Geometric distribution with Mean μ=1/p = 500 hours.
- System S is in state Down; then moves to state Up in one-step, with Probability p_{01}= q = 0.033
  - A Geometric distribution, with Mean μ=1/q = 30 hours.
- Every step (time period to transition) is an hour.
- The Geometric Distribution is the Discrete counterpart of the Continuous Exponential

Transition Probability Matrix P

$$\begin{array}{ccc} States & 0 & 1 \\ 0 & (1-q & q) \\ 1 & (p & 1-p) \end{array} = \begin{array}{ccc} States & 0 & 1 \\ 0 & (0.967 & 0.033) \\ 1 & (0.002 & 0.998) \end{array}$$

Entries of Matrix P = (p_{ij}) correspond to Markov Chain's one-step transition probabilities. Rows represent every state that system S can be in, at any time T. Columns represent every other state that system S can go into, in one step (i.e. state where S will be, at time T+1). Unit of time T, can be made as small or as large, as necessary. Transition Probabilities (p_{ij}) can be obtained *empirically*.

We obtain probability of S moving from state Up to Down, in Two Hours (steps):

$$P^2 = \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix}^2 = \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix} \times \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix}$$

$$= \begin{bmatrix} (1-q)^2 + pq & q(1-q) + q(1-p) \\ p(1-q) + p(1-p) & pq + (1-p)^2 \end{bmatrix} = \begin{bmatrix} p_{oo}^{(2)} & p_{01}^{(2)} \\ p_{10}^{(2)} & p_{11}^{(2)} \end{bmatrix}$$

$$\Rightarrow p_{10}^{(2)} = p_{10}p_{00} + p_{11}p_{10} = p(1-q) + (1-p)p$$

The probability that S will go down in two hours (steps) is:

$$p_{10}^{(2)} = p(1-q) + (1-p)p = 0.003$$

Other useful Markov results:

- If $p_{10}^{(2)} = 0.003 \Rightarrow p_{11}^{(2)} = 1 - p_{10}^{(2)} = A(T) = 0.993$
  – A(T) system Availability, after T=2 hours operation
- Prob. of moving from state 1 to 0, in 10 steps:
  – $(P)^{10} \Rightarrow p_{10}^{(10)} = 0.017$; calcs include that S could have gone Down or Up, then restored again, several times.
- For a sufficiently large **n** (long run) and two-states:

$$Limit_{n\to\infty} P^n = Limit_{n\to\infty} \{ \frac{1}{p+q} \begin{bmatrix} p & q \\ p & q \end{bmatrix} + \frac{(1-p-q)^n}{p+q} \begin{bmatrix} q & -q \\ -p & p \end{bmatrix} \} = \begin{bmatrix} p/(p+q) & q/(p+q) \\ p/(p+q) & q/(p+q) \end{bmatrix}$$

Example: Up = q/(p+q) = 0.943 ; Down = p/(p+q) = 0.057

Markov Model for Redundant system

- A Redundant System is one, composed of Two Identical Devices, in Parallel.
- The System is maintained and can function at a Degraded level (i.e. with only one unit UP)
- The System has now Three States: 0, 1, 2:
  - State 0, the <u>Down state</u>; both units are DOWN
  - State 1, the <u>Degraded state</u>; only one unit is UP
  - State 2, the <u>UP state</u>; both units OPERATING

Markov Model system representation

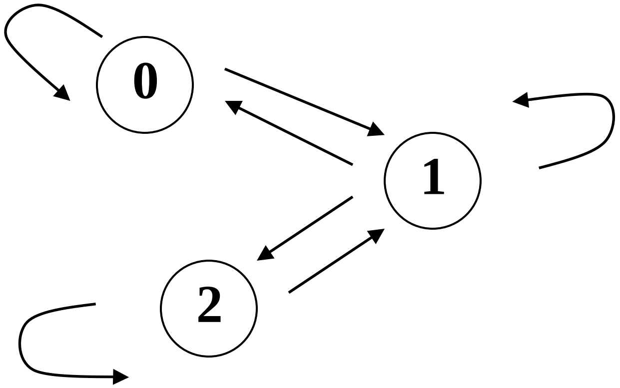
$$p_{01} = P\{X(T) = 1 \mid X(T-1) = 0\} = q$$

$$p_{10} = P\{X(T) = 0 \mid X(T-1) = 1\} = p$$

$$p_{12} = P\{X(T) = 2 \mid X(T-1) = 1\} = q$$

$$p_{21} = P\{X(T) = 1 \mid X(T-1) = 2\} = 2p$$

$$p_{ii} = P\{X(T) = i \mid X(T-1) = i\} = 1 - \sum_{j \neq i} p_{ij}$$



Operational Conditions

- Every step (hour) T is an independent trial
- Success Prob. p_{ij} corresponds to a transition from current state ‘i’ into state ‘j’ = 0,1,2
- Distribution of every change of state is the Geometric (Counterpart of the Exponential)
- Mean time to accomplishing such change of state is: $\mu = 1/p_{ij}$
- Time units are arbitrary; p_{ij} can be estimated

Transition Probability Matrix P:

$$P = \begin{array}{cccc} States & 0 & 1 & 2 \\ 0 & p_{00} & p_{01} & p_{02} \\ 1 & p_{10} & p_{11} & p_{12} \\ 2 & p_{20} & p_{21} & p_{22} \end{array} = \begin{array}{cccc} States & 0 & 1 & 2 \\ 0 & 1-q & q & 0 \\ 1 & p & 1-p-q & q \\ 2 & 0 & 2p & 1-2p \end{array}$$

As before, the probability of being in state "j" after "n" steps, given that we started in some state "i" of S, is obtained by raising matrix P to the power "n", and then looking at entry p_{ij} of the resulting matrix P^n.

Numerical Example

- Probability p of either of the two units failing
  - in the next hour, is $p = 0.002$
- Probability q of the repair crew completing
  - a maintenance job in the next hour is $q = 0.033$
- Only one failure is allowed
  - in each unit time period,
- and only one repair can be undertaken
  - in each unit time period

Probability that a degraded system (in State 1) remains degraded, after two more hours of operation:

- Sum probabilities corresponding to 3 events
  - the system status has never changed
  - one unit repaired but another fails during 2nd hour
  - remaining unit fails in the first hour (system goes down), but a repair is completed in the 2nd hour

Numerical Example:

$$P_{11}^{2} = [P \times P]_{11} = p_{11}^{(2)} = p_{10}p_{01} + p_{11}p_{11} + p_{12}p_{21}$$

$$= pq + (1 - p - q)^{2} + 2pq$$

$$= 0.002 \times 0.033 + (1 - 0.035)^{2} + 2 \times 0.002 \times 0.033$$

$$P_{11}^{2} = 0.9314$$

The probability that a system, in degraded state, is still in degraded state, after two more hours working, is: $P^{2}_{11} = 0.9314$

Mean time μ that the system S spends in the Degraded State

- System S can change to Up or Down
  - with probabilities p and q, respectively
- S will remain in the Degraded State
  - with probability 1- p- q (i.e. no change)
- On average, S will spend a "sojourn" of
  - length $1/(p + q) = 1/0.035 = 28.57$ hrs
  - in the Degraded State, before moving out.

Availability at time T

- A(T) = P{S is Available at T}
- System S is not Down at time "T"
  - Then, S can be either Up, or Degraded
- A(T) depends on the initial state of S
- Find Prob. S is "Degraded Available" at T
  - given that S was Degraded (initially 1) at T=0

$$p_{10}^{(T)} + p_{11}^{(T)} + p_{12}^{(T)} = 1 \Rightarrow p_{11}^{(T)} = 1 - p_{10}^{(T)} - p_{12}^{(T)}$$

$$A(T) = P\{X(T) = 1 \mid X(0) = 1\} = p_{11}^{(T)} = 1 - p_{10}^{(T)} - p_{12}^{(T)}$$

State Occupancies

- Long Run Averages of system *sojourns*
- Asymptotic probabilities of system S being
  - in each one of its possible states at any time T
- Or the percent time S spent in these states
  - Irrespective of the state S was in, initially.
- Results are obtained by considering
  - Vector Π of the "Long Run" probabilities:

Characteristics of Vector Π

$$\Pi = Limit_{T\to\infty}(\Pr ob\{X(T)=0\}, \Pr ob\{X(T)=1\}, \Pr ob\{X(T)=2\})$$

Vector Π fulfills two important properties:

$$(1):\Pi\times P=\Pi;(2)\sum\Pi_i=1; with:\Pi_i=Limit_{T\to\infty}\Pr ob\{X(T)=i\}$$

Π × P = Π (Vector Π times matrix P equals Π) defines a *system of linear equations*, "normalized" by the 2nd property.

$$\Pi\times P=\left(\Pi_0,\Pi_1,\Pi_2\right)\times\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}=\left(\Pi_0,\Pi_1,\Pi_2\right)$$

$$with:\sum_i\Pi_i=\Pi_0+\Pi_1+\Pi_2=1$$

Numerical Example

$$\Pi \times P = (\Pi_0, \Pi_1, \Pi_2) \times \begin{bmatrix} 0.967 & 0.033 & 0 \\ 0.002 & 0.965 & 0.033 \\ 0 & 0.004 & 0.996 \end{bmatrix} = (\Pi_0, \Pi_1, \Pi_2)$$

$$\Rightarrow \begin{cases} 0.967\Pi_0 + 0.002\Pi_1 = \Pi_0 \\ 0.033\Pi_0 + 0.965\Pi_1 + 0.004\Pi_2 = \Pi_1; with: \sum_i \Pi_i = \Pi_0 + \Pi_1 + \Pi_2 = 1 \\ 0.033\Pi_1 + 0.996\Pi_2 = \Pi_2 \end{cases}$$

Solution of system yields Long Run Occupancy rates:

$$\Pi = (\Pi_0, \Pi_1, \Pi_2) = (0.0065, 0.1074, 0.8861)$$

Interpretation of these results:

- $\Pi_2 = 0.8861$ indicates that system S
  - is operating at full capacity 88.6% of the time.
- $\Pi_1 = 0.1074$ indicates that system S
  - is operating at Degraded capacity 10.7% of the time.
- Π_0: probability corresponding to State 0 (Down)
  - is associated with S being Unavailable (= 0.0065)
- "Long Run" System Availability is given by:
  - $A = 1 - \Pi_0 = 1 - 0.0065 = 0.9935$

Calculate Expected Times

- For System S to go Down, if initially
  - S was Up (denoted V_2), or Degraded (V_1)
- Or the average time System S spent in each
  - of these states (1, 2) before going "Down".
- Assume Down is an "***absorbing state***"
  - one that, once entered, can never be left
- Solve a system of equations leading to
  - all such possible situations.

Numerical Example:

One step, at minimum (initial visit), before system S goes Down.
If S is not absorbed then, system S will move on to any of other, non-absorbing (Up, Degraded) states with corresponding probability, and then the process restarts:

$$V_1 = 1 + p_{11}V_1 + p_{12}V_2 = 1 + 0.965V_1 + 0.033V_2$$
$$V_2 = 1 + p_{21}V_1 + p_{22}V_2 = 1 + 0.004V_1 + 0.996V_2$$

Solving, yields the Average times until system S goes down :
V_1= 4625 (if starting in state Degraded) and
V_2 = 4875 (if starting in state Up).

Model Performance Comparison

- The initially non-maintained system version
  - would work an Expected $3/2\lambda = 3/0.004 = 750$
  - hours in Up state, before going Down (Ref. 7).
- If system maintenance is now possible, and S can operate in a Degraded State:
  - results in *additional:* $\mu/2\lambda^2 = 0.033/2 \times 0.002^2 = 4125$
  - hours of Expected Time before going Down (from Up)
- The new Total Expected Time, is the Sum of the Two Expected times to failures:

  $= 3/2\lambda + \mu/2\lambda^2 = 750 + 4125 = 4875$

Practical Example for Increasing Availability

- Assume your System has currently
- An Achieved Availability of 85%
- This Availability is unacceptably LOW
- And requires to be Improved

$$A = \frac{MTBF}{MTBF + MTTR} = \frac{85}{85 + 15} = 0.85 \approx 85\%$$

Economically Increasing System Availability

- Assume MTTR is largely affected by delays:
  - Waiting for a specialist mechanic
  - Waiting for a special spare part
- Assume Availability HAS to be <u>at least</u> 90%:
  - We can hire additional specialists or mechanics
  - We can increase the warehouse parts inventory
- Assume such would reduce MTTR to 8 units:

Therefore, the New System Availability is:

$A = 85/(85+8) = 85/93 = 0.914 \sim 91.4\%$

Conclusions

- Availability is the ratio of:
  - *Up.Time* to *Cycle.Time*
- Hence, we can enhance Availability by:
  - Increasing the device or system Life (R)
  - Decreasing/Improving maintenance time
  - Simultaneously, doing both above.
- Decreasing maintenance is usually:
  - Easier and/or Cheaper.

Juarez Lincoln Marti Int’l Ed. Project

- The JLM International Project develops programs to support Higher Education in Iberoamerica
- Its Web Page is: http://web.cortland.edu/matresearch/
- A quick overview of the Project is in PPT:
  - http://web.cortland.edu/romeu/JuarezUscots09.pdf
- JLM Project Sponsors the Quality, Reliability and Industrial Statistics Institute Web Site, accessible from its Web Page.

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About the Author

Jorge Luis Romeu has over 40 years applying statistical and operations research methods to HW/SW reliability, quality and industrial engineering. Romeu retired Emeritus from SUNY, where he taught mathematics, statistics and computers. He was a Fulbright Senior Specialist, at universities in Mexico (1994, 2000 and 2003), Dominican Republic (2004), and Ecuador (2006). He created and directs the Juarez-Lincoln-Marti Int'l Ed. Project. Romeu is also an Adjunct Professor, Syracuse University, where he teaches statistics, quality and operations research courses. He worked as a Senior Engineer for IIT Research Institute and the RIAC (Reliability Information Analysis Center). Romeu is lead author of A Practical Guide to Statistical Analysis of Materials Property Data. He has developed and teaches many workshops and training courses for practicing engineers and statistics faculty, and has published over forty articles on applied statistics and statistical education. He obtained the Saaty Award for the Best Applied Statistics Paper in American Journal of Mathematics and Management Sciences (AJMMS), in 1997 and 2007. Romeu holds a Ph.D. in Operations Research, is a Chartered Statistician Fellow of the Royal Statistical Society, member of the American Statistical Association, ans Senior Member of the American Society for Quality. He holds ASQ certifications in Quality and Reliability and is Past Regional Director of ASQ Region II. For more information, visit his web site http://web.cortland.edu/romeu