

Example of the Comparison of a Half Fraction 2³ DOE, with its Full Design Version:

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Full Data: (Thee replications for each of the eight treatments of a 2³)

Y1	Y2	Y3
-2.50	-2.42	1.72
3.56	0.73	6.87
-1.71	-0.75	0.72
10.98	11.64	13.50
10.52	4.12	8.61
14.77	18.00	13.57
11.19	12.09	10.00
19.71	15.02	20.19

Full 2³ Design Matrix:

A	B	C	AB	AC	BC	D
-1	-1	-1	1	1	1	-1
1	-1	-1	-1	-1	1	1
-1	1	-1	-1	1	-1	1
1	1	-1	1	-1	-1	-1
-1	-1	1	1	-1	-1	1
1	-1	1	-1	1	-1	-1
-1	1	1	-1	-1	1	-1
1	1	1	1	1	1	1

Factorial Fit: Y1 versus A, B, C

Estimated Effects and Coefficients for Y1 (coded units)

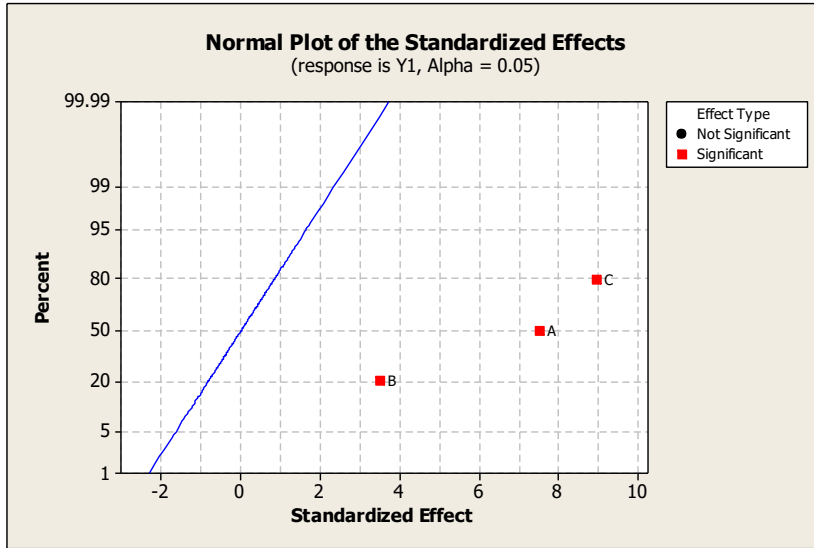
Term	Effect	Coef	SE Coef	T	P
Constant		8.339	0.5373	15.52	0.000
A	8.079	4.039	0.5373	7.52	0.000
B	3.753	1.876	0.5373	3.49	0.002
C	9.622	4.811	0.5373	8.95	0.000

S = 2.63213 PRESS = 199.529
R-Sq = 88.16% R-Sq(pred) = 82.95% R-Sq(adj) = 86.38%

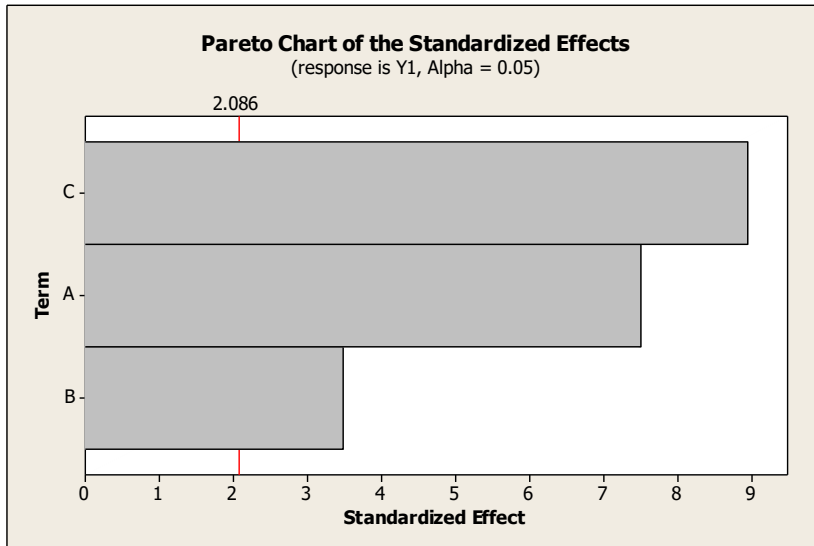
Analysis of Variance for Y1 (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	1031.56	1031.56	343.854	49.63	0.000
Residual Error	20	138.56	138.56	6.928		
Lack of Fit	4	51.04	51.04	12.760	2.33	0.100
Pure Error	16	87.52	87.52	5.470		
Total	23	1170.12				

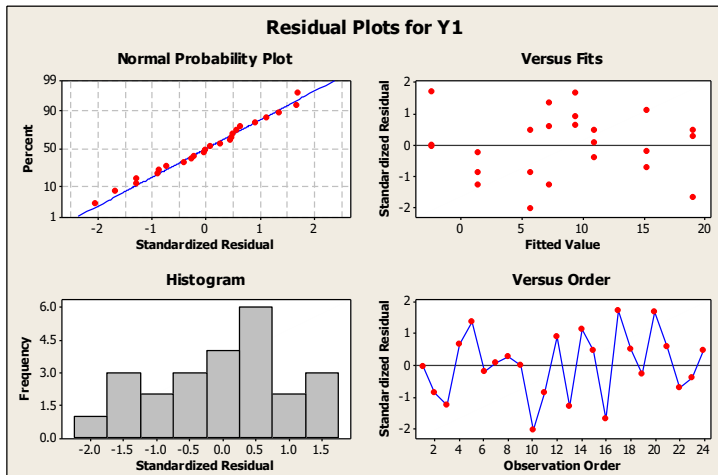
Effects Plot for Y1



Effects Pareto for Y1



Residual Plots for Y1



We now implement a Half Fraction, using a 2² Full Design and making C=AB (selection highlighted)

Full 2² Design Matrix:

Treat	A	B	AB
1	-1	-1	1
a	1	-1	-1
b	-1	1	-1
ab	1	1	1

Taking the corresponding values from the 2³ Matrix above: (Generator: C=AB => I=ABC)

A	B	C=AB	Y1	Y2	Y3
-1	-1	1	10.52	4.12	8.61
1	-1	-1	3.56	0.73	6.87
-1	1	-1	-1.71	-0.75	0.72
1	1	1	19.71	15.02	20.00

Factorial Fit: Response versus A, B, C=AB

Estimated Effects and Coefficients for Response

Term	Effect	Coef	SE Coef	T	P
Constant		7.283	0.7846	9.28	0.000
A	7.397	3.698	0.7846	4.71	0.002
B	3.097	1.548	0.7846	1.97	0.084
C=AB	11.427	5.713	0.7846	7.28	0.000

S = 2.71800 PRESS = 132.976
R-Sq = 90.82% R-Sq(pred) = 79.34% R-Sq(adj) = 87.38%

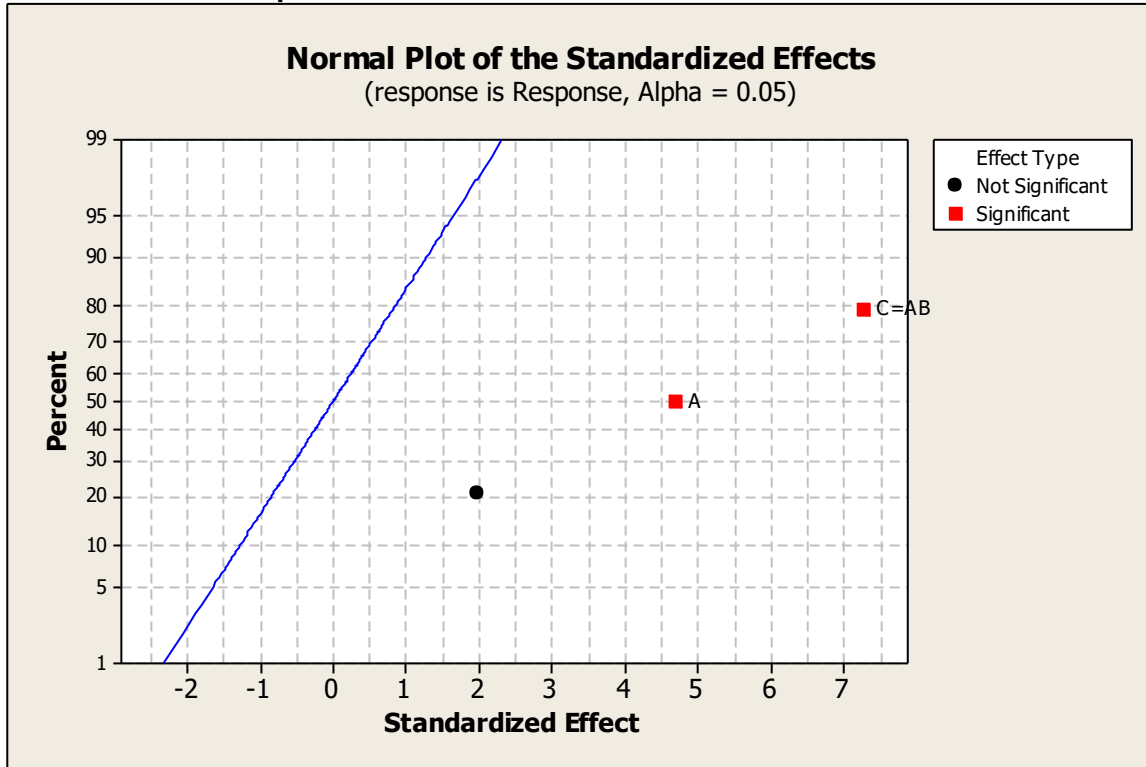
Analysis of Variance for Response (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	584.606	584.606	194.869	26.38	0.000
Residual Error	8	59.100	59.100	7.388		
Pure Error	8	59.100	59.100	7.388		
Total	11	643.706				

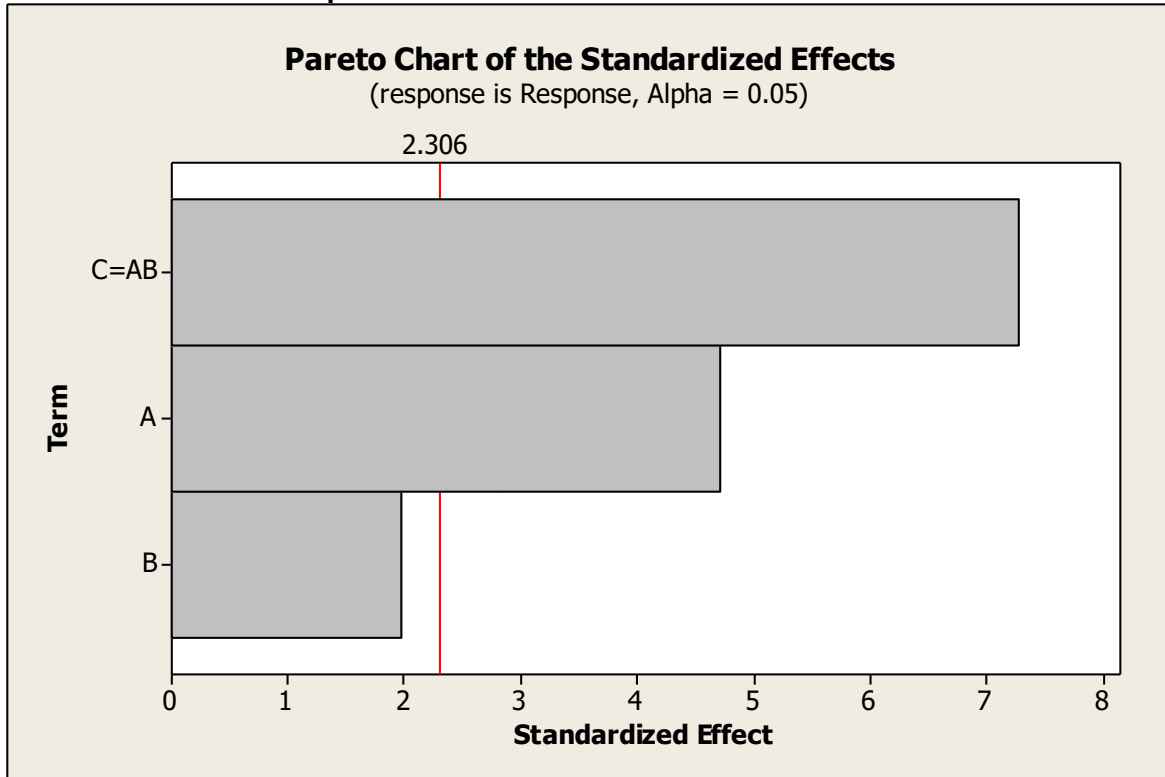
Alias Structure

I = A*B*(C=AB)
A = B*(C=AB)
B = A*(C=AB)
C or AB = A*B

Effects Plot for Response

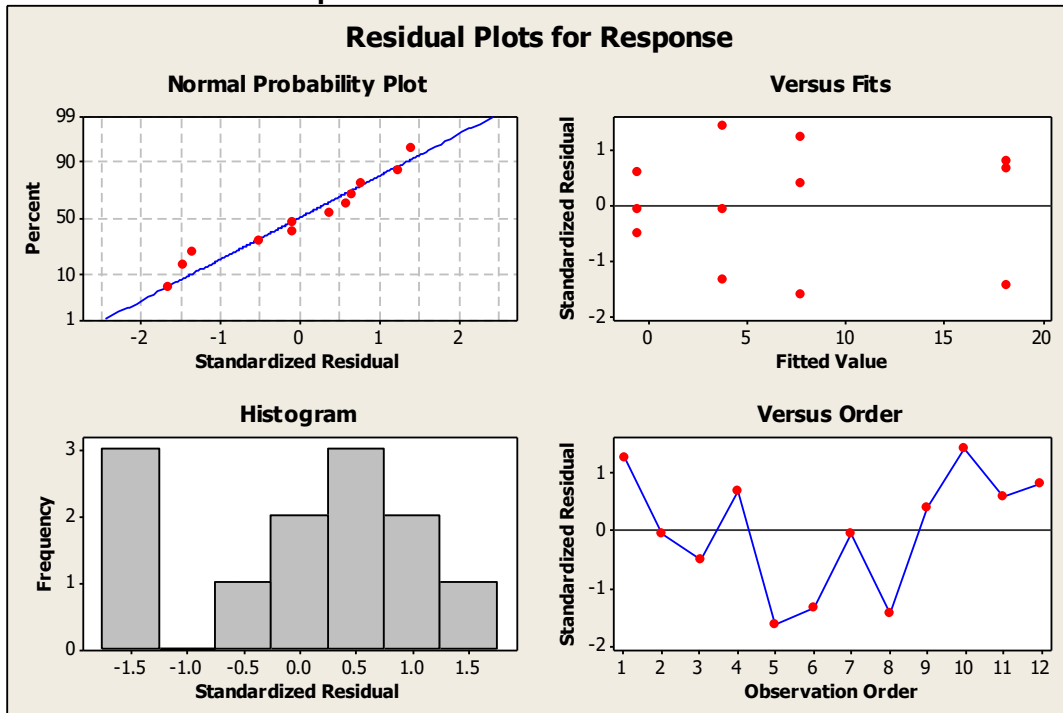


Effects Pareto for Response

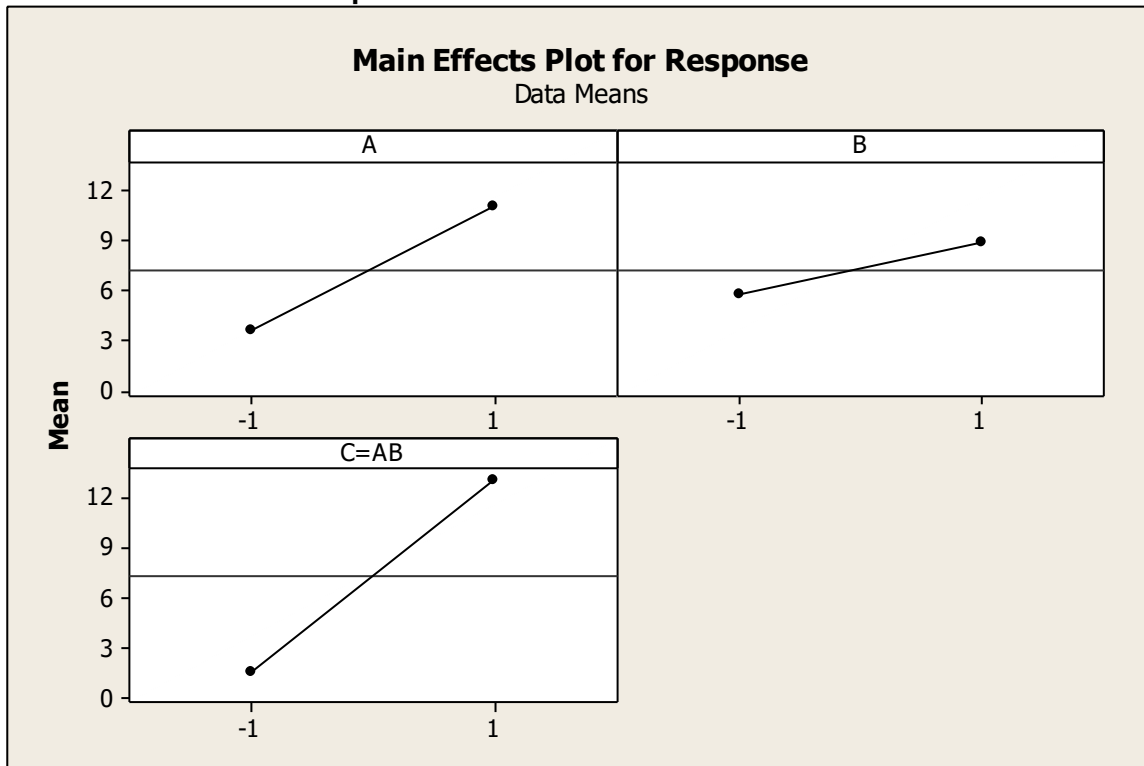


$$\text{Standardized Effect} = \text{Effect} / \text{Std-Dev}(\text{Effect})$$

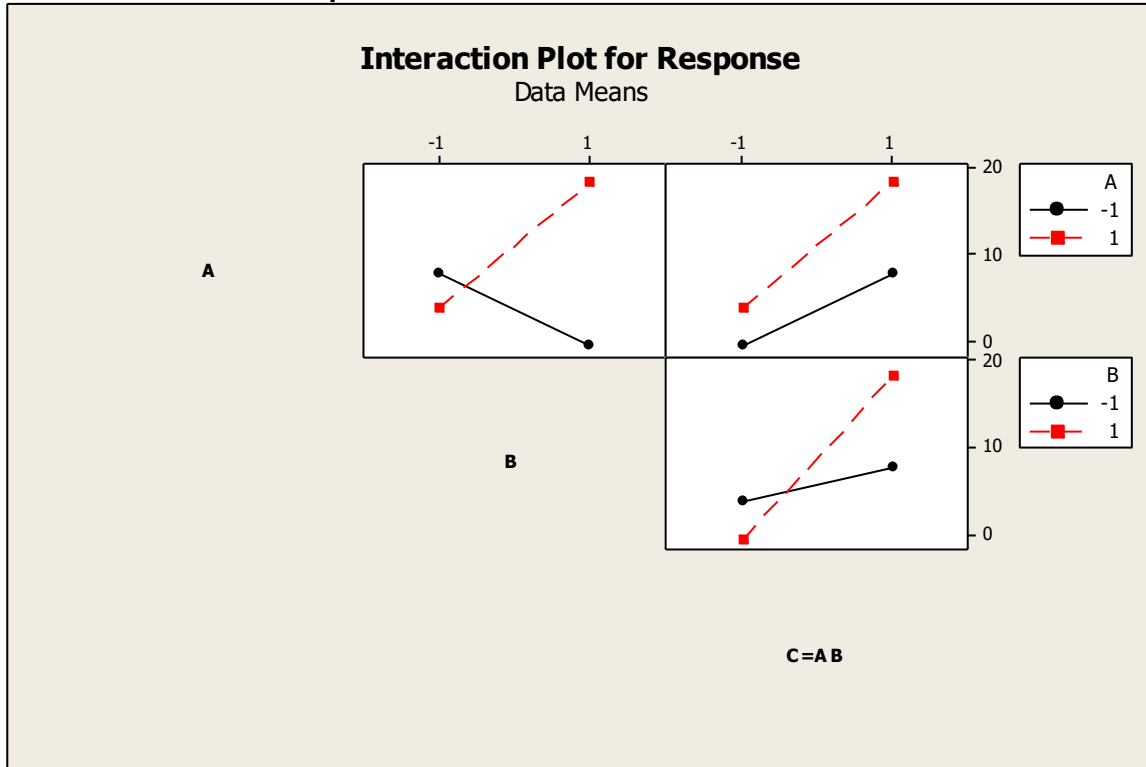
Residual Plots for Response



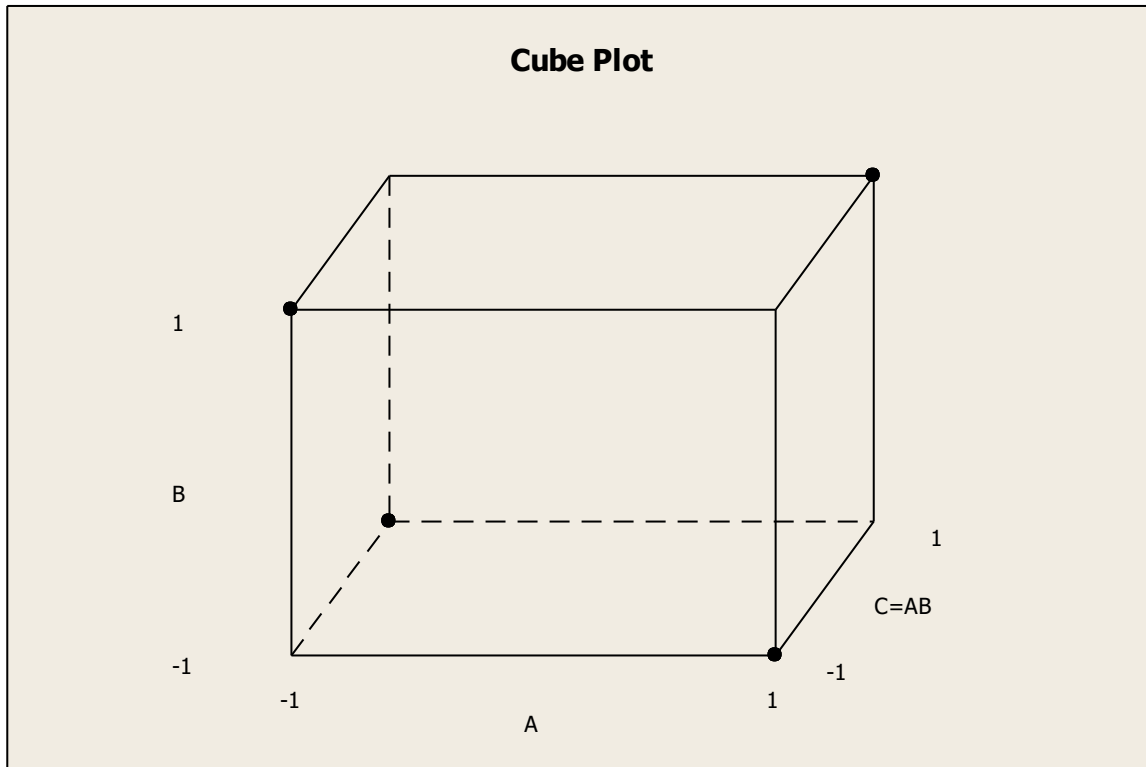
Main Effects Plot for Response



Interaction Plot for Response



Cube Plot



Corners in Black constitute the four rows used in the Half Fraction

Comparison:

Factor	Full DOE	p-val	Half Fraction	p-val
A	8.07	0.0	7.39	0.002
B	3.75	.06	3.09	0.084
AB	9.62	.003	11.43	0.000

Conclusions:

We can see how effects of A and B are closely estimated in both analyses; but the effect of C = AB in the Half Fraction, is further off. P-values are commensurate.

Loss of precision is the price to pay for the reduction in experimentation.