Overview of Fractional Factoril DOE for Reliability Improvements

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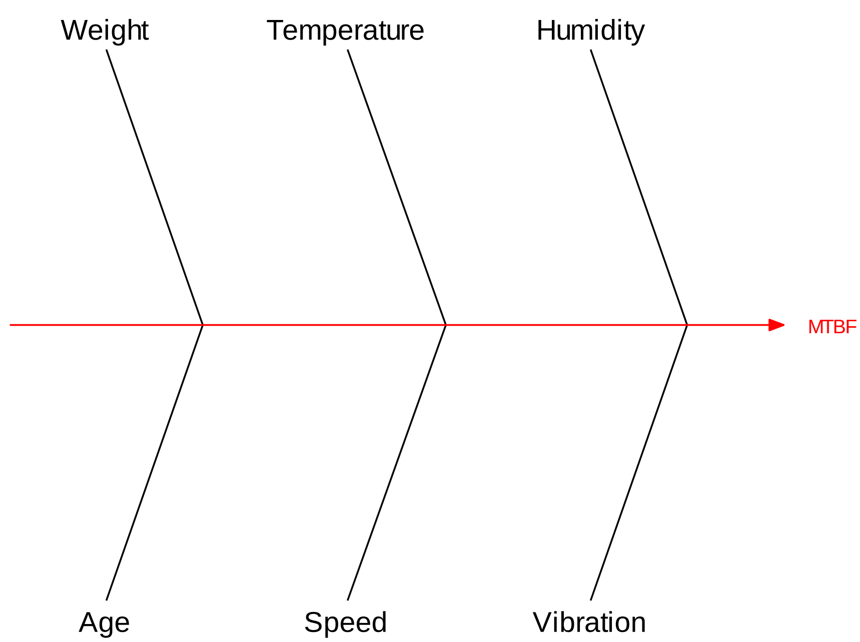
Assume we want to improve Reliability or other important Quality characteristic, by increasing positive and reducing negative factor effects, that influence its performance.

For example, assume we want to increase MTBF (or decrease MTTR) of a device, to improve its overall Availability. We need to first identify which factors are affecting the performance measure, and then investigate what effects, if any, these factors have on it. Brainstorming, we can identify candidate factors that we can then put into a Fishbone chart. If our device is a vehicle, and we want to increase its MTBF, we may have:

Candidate Factors Affecting MTBF



We can assess their impact by implementing a Designed Experiment (DOE) and then analyzing the collected data. Often, however, the number of factors analyzed, which determine the number of runs to implement, may make an experiment extremely costly or time consuming. We need to reduce the number of runs, in order to make it feasible. We can achieve this reduction by implementing “Fractional” versions of the Experimental Factorial Designs. Let’s give an example.

Assume we are restricted to, at most, eight experimental runs. We select the four most important of the candidate factors, and carry out a four-factor Half Fraction Factorial

DOE. We select Environmental Temperature (A), Humidity (B), Speed (C) and Weight (D). We use two conveniently selected levels for each factor: low (-) and high (+). This yields a ½*(2^4) = ½*16 = 8 run Factorial design; one that we can now afford. For, we have reduced the number of runs from 16 (complete factorial) to only eight.

Factor	Low (-)	High (+)
Temp	40 F	80 F
Humid	30%	60%
Speed	30 mph	50 mph
Weight	1 Ton	2 Tons

The experimental results of the implementation of the eight runs of such experiment, for response MTBF, is given in the table below (letters A, B, C, D are used for Factor names and + and – for low and high levels, respectively). For, the values for each Factor (e.g. Temperature, Humidity, etc.) are processed by the DOE algorithm in "coded" form. Each Factor at two levels defines a Range, with a Minimum, a Midpoint and a Maximum. For example, Minimum for Factor Temperature is 40 F and becomes -1, and Maximum is 80 F and becomes +1. Its Midpoint is 60. Any intermediate coded value is then interpolated between -1 and +1, using the formula:

$$\text{Coded} = (\text{RealValue} - \text{Midpoint}) / \text{HalfRange}$$

For example, at the four lower levels of Temperature, Humidity, Speed and Weight (i.e. 40 F, 30%, 30 mph and 1 Ton) the experimental MTBF was 66.63 hours. We enter such information in the Design of Experiments matrix below:

Design of Experiments Analysis:
Factorial Experiments 2^3 (No replications)

Run	**A**	**B**	**C**	**D**	**Responses**
1	-1	-1	-1	-1	66.63
2	1	-1	-1	1	77.25
3	-1	1	-1	1	50.25
4	1	1	-1	-1	66.91
5	-1	-1	1	1	60.31
6	1	-1	1	-1	69.98
7	-1	1	1	-1	56.46
8	1	1	1	1	74.88
SumY+	289.02	248.50	261.63	268.73	
SumY-	233.65	274.17	261.04	253.94	
AvgY+	72.26	62.13	65.41	67.18	
AvgY-	58.41	68.54	65.26	63.49	
Effect	**13.84**	**-6.42**	**0.15**	**3.70**	

The numerical results for our ½*(2^4) = ½*16 = 8 run ½ Half Fractional Factorial is thence obtained. Main Effects are obtained by subtracting Average(+) – Average(-). These are the "Main Effects" of the Four Factors A, B, C and D.

These values constitute their (positive or negative) contribution to the *Response* MTBF, as each factor moves from the low to the high level. For example, Factor A has a positive effect (i.e., value 13.842) on the response MTBF.

There is a price to pay, when using a Half Fraction Design, as opposed to a Full Design. It is that Factor Effects are "confounded". That is, certain main effects and interactions are inextricably mixed, so one cannot tell whether the "effect" is from one, or the other. This is due to the fact that we have substituted the Interactions by Factors.

In the present case, we substituted the Triple Interaction ABC by Factor D: D=ABC. The *governing relation* is I=ABCD. And all possible *confounded effects* are thus derived.

For example, Factor A in column 1, is "confounded" with interaction "BCD". Therefore, we cannot say whether this effect is due to A, or to BCD (or perhaps to both). However, higher order interactions are usually not very significant. We can also have some prior knowledge about the vehicle under analysis, to resolve this issue. So, confounding of effects is something that can be handled with success.

We still have to assess, statistically, the results obtained. Such assessment is obtained by defining an "acceptable risk alpha" of stating that a Factor is significant, when it is not. In our case, we would like this risk to be alpha = 10%. To assess this situation we use the Fisher "F" percentile, for the corresponding Degrees of Freedom (in this case, DF are 1 and 3). The percentile $F(1, 3; \alpha = 0.05) = 5.54$ serves as comparison for the ratios of the corresponding Factor Mean Square and the Error Mean Square (given in the F-Ratio column of this sub-section).

For example, for Factor A (Temperature), the only significant one here, the Ratio:

$$F = 383.202/41.281 = 9.283 > 5.54$$

Hence, we can state, with 90% confidence, that the environmental Temperature has a positive influence in extending the reliability (MTBF). Experiments show vehicles work longer times without experimenting failure, when operating in higher temperatures (80s F), than when operating in lower temperatures (i.e. colder weather: 40 F).

The other three factors, humidity, speed at which vehicles operate, and weight (cargo), do not seem to have a significant effect on reliability (MTBF). For, none of its F-Ratios is larger than percentile $F(1, 3; \alpha = 0.05) = 5.54$.

Finally, we need the *Response equation* for MTBF. In our example, this equation is:

$$Y = 65.334 + 6.921 * \text{Temperature} + 0 * \text{Humidity} + 0 * \text{Speed} + 0 * \text{Weight}$$

Remember, however, that the values used for the Factors (Temperature, Humidity, etc.) were given to the DOE in *coded form.* Therefore, they have to be entered in the *Response* equation also in "coded" form. We need to convert them back. For our example:

Coded-T = (Temp-60) / 20 ; Coded-H = (Hum-45) / 15 ; etc.

The *Response* equation then provides a point estimate of MTBF, for a specific setting. If we want an estimate MTBF for vehicle operation, in an environment temperature of 70 F, of humidity 50%, when the vehicle is running at 40mph, and is carrying 1.5 tons, we first convert these values to coded form:

Coded-T = (70-60) / 20 = 10/20; Coded-H = (50-45) / 15 = 5/15; etc

Hence, we enter 0.5 for Temperature 70, 0.33 for Humidity 50% and Zero for 40 mph and 1.5 Tons, respectively, in the model equation:

$$MTBF = 65.334 + 6.921 * 0.5 + 0 * (0.33) + 0 * 0 + 0 * 0$$

$$= 65.334 + 6.921 * 0.5 = 68.79 \text{ hours}$$

An interpretation of these results is: *Increasing variables with positive coefficients and minimizing those with negative coefficients will optimize the design*.

This means that "statistically significant" coefficients are placed in the equation, and "non-significant" coefficients, appear with value Zero (i.e. have no influence). This example provides three non-significant Factors: humidity, speed and weight, at level Zero, and one significant Factor: Temperature (6.921)

Summarizing, DOE can help reliability engineers analyze their experiments and use their statistical results to optimize (or improve) their design (or operation).