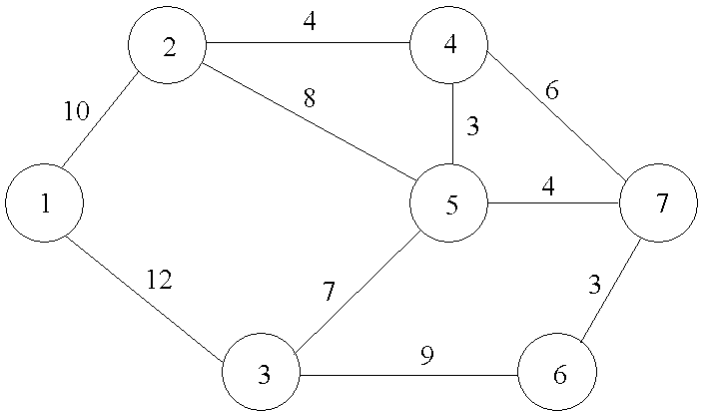
MFE411/511 – Intro to Management Science

Group No._____ Date:_____

I. Military Intelligence has rated the road segments between Kabul (1) and Herat (7) according to their risk of attack from the Taliban, as indicated in the Graph below:



Find the "shortest path" from 1 to 7, i.e. *the one with the smallest total risk for a military convoy,* to make that move. Include all equations, the Solver program, its complete solution, and its interpretation.

ANSWER:

$$\text{Min} \quad 10X_{12} + 12X_{13} + 4X_{24} + 8X_{25} + 7X_{35} + 9X_{36} + 4X_{42} + 3X_{45} + 6X_{47} + 8X_{52} + 7X_{53} + 3X_{54} + 4X_{57} + 9X_{63} + 3X_{67}$$

s.t.
$$X_{12} + X_{13} = 1$$
$$-X_{12} + X_{24} + X_{25} - X_{42} - X_{52} = 0$$
$$-X_{13} + X_{35} + X_{36} - X_{53} - X_{63} = 0$$
$$-X_{24} + X_{42} + X_{45} + X_{47} - X_{54} = 0$$
$$-X_{25} - X_{35} - X_{45} + X_{52} + X_{53} + X_{57} = 0$$
$$-X_{36} + X_{63} + X_{67} = 0$$
$$X_{47} + X_{57} + X_{67} = 1$$
$$X_{ij} \geq 0 \text{ for all i,j}$$

Lingo Solution:

Model:
Min = $3*X_{12} + 2*X_{13} + 4*X_{24} + 5*X_{25} + 3*X_{34} + 7*X_{35} + 4*X_{42} + 3*X_{43} + 6*X_{46} + 5*X_{52} + 7*X_{53} + 2*X_{56}$;
!Subject to;
$X_{12} + X_{13} = 1$;
$-X_{12} + X_{24} + X_{25} - X_{42} - X_{52} = 0$;
$-X_{13} + X_{34} + X_{35} - X_{43} - X_{53} = 0$;
$-X_{24} - X_{34} + X_{42} + X_{43} + X_{46} = 0$;
$-X_{25} - X_{35} + X_{52} + X_{53} + X_{56} = 0$;
$X_{46} + X_{56} = 1$;
End

Global optimal solution found.	
Objective value:	10.00000
Infeasibilities:	0.000000
Total solver iterations:	3
Elapsed runtime seconds:	0.09
Model Class:	LP
Total variables:	12
Nonlinear variables:	0
Integer variables:	0
Total constraints:	7
Nonlinear constraints:	0
Total nonzeros:	36
Nonlinear nonzeros:	0

Variable	Value	Reduced Cost
X12	1.000000	0.000000
X13	0.000000	0.000000
X24	0.000000	3.000000
X25	1.000000	0.000000
X34	0.000000	1.000000
X35	0.000000	1.000000
X42	0.000000	5.000000
X43	0.000000	5.000000
X46	0.000000	0.000000
X52	0.000000	10.00000
X53	0.000000	13.00000
X56	1.000000	0.000000

Row	Slack or Surplus	Dual Price
1	10.00000	-1.000000
2	0.000000	-3.000000
3	0.000000	0.000000
4	0.000000	-1.000000
5	0.000000	1.000000
6	0.000000	5.000000
7	0.000000	-7.000000

Ranges in which the basis is unchanged:

Objective Coefficient Ranges:

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
X12	3.000000	1.000000	5.000000
X13	2.000000	5.000000	1.000000
X24	4.000000	INFINITY	3.000000
X25	5.000000	1.000000	5.000000
X34	3.000000	INFINITY	1.000000
X35	7.000000	INFINITY	1.000000
X42	4.000000	INFINITY	5.000000

X43	3.000000	INFINITY	5.000000
X46	6.000000	5.000000	1.000000
X52	5.000000	INFINITY	10.00000
X53	7.000000	INFINITY	13.00000
X56	2.000000	1.000000	5.000000

Righthand Side Ranges:

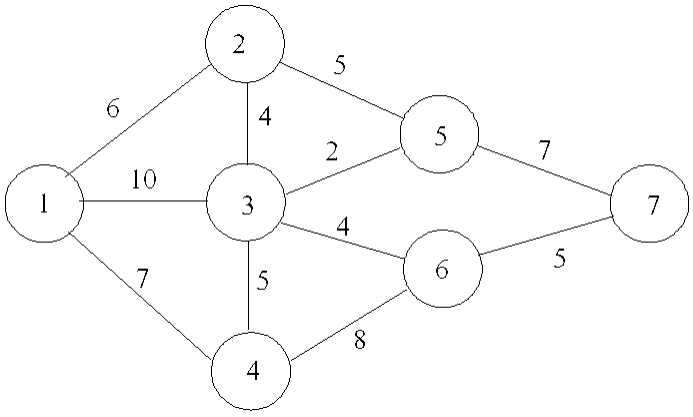
Row	Current RHS	Allowable Increase	Allowable Decrease
2	1.000000	0.000000	0.000000
3	0.000000	0.000000	0.000000
4	0.000000	0.000000	0.000000
5	0.000000	0.000000	0.000000
6	0.000000	0.000000	0.000000
7	1.000000	0.000000	0.000000

II There is a need to transport the largest number of troops possible from Kabul (1) to Kandahar (7). The difficulties of the terrain, the conditions of the roads, and the attack risks are used to estimate the number of troop transports (i.e. trucks) per hour that can traverse these segments (given below).:

Segment	Transp./Hr
1 to 2	6
1 to 3	10
1 to 4	7
2 to 3	4
2 to 5	5
3 to 4	5
3 to 5	2
3 to 6	4
4 to 6	8
5 to 7	7
6 to 7	5

Based on the above data, draw the network graph, develop the algebraic equations for this Max Flow problem, and use Solver to obtain the optimal troop flow from Kabul to Kandahar.

ANSWER:



Min $6X_{12} + 10X_{13} + 7X_{14} + 4X_{23} + 5X_{25} + 4X_{32} + 5X_{34} + 2X_{35} + 4X_{36} + 5X_{43} + 8X_{46} + 5X_{52} + 2X_{53} + 7X_{57} + 4X_{63} + 8X_{64} + 5X_{67}$

s.t.
$$X_{12} + X_{13} + X_{14} = 1$$
$$-X_{12} + X_{23} + X_{25} - X_{32} - X_{52} = 0$$
$$-X_{13} - X_{23} + X_{32} + X_{34} + X_{35} + X_{36} - X_{43} - X_{53} - X_{63} = 0$$
$$-X_{14} - X_{34} + X_{43} + X_{46} - X_{64} = 0$$
$$-X_{25} - X_{35} + X_{52} + X_{53} + X_{57} = 0$$
$$-X_{36} - X_{46} + X_{63} + X_{64} + X_{67} = 0$$
$$X_{57} + X_{67} = 1$$
$$X_{ij} \geq 0 \text{ for all i,j}$$

Lingo Solution:

Model:
Min = $6*X_{12} + 10*X_{13} + 7*X_{14} + 4*X_{23} + 5*X_{25} + 4*X_{32} + 5*X_{34} + 2*X_{35} + 4*X_{36} + 5*X_{43} + 8*X_{46} + 5*X_{52} + 2*X_{53} + 7*X_{57} + 4*X_{63} + 8*X_{64} + 5*X_{67}$;
!Subject to;
$X_{12} + X_{13} + X_{14} = 1$;
$-X_{12} + X_{23} + X_{25} - X_{32} - X_{52} = 0$;
$-X_{13} - X_{23} + X_{32} + X_{34} + X_{35} + X_{36} - X_{43} - X_{53} - X_{63} = 0$;
$-X_{14} - X_{34} + X_{43} + X_{46} - X_{64} = 0$;
$-X_{25} - X_{35} + X_{52} + X_{53} + X_{57} = 0$;
$-X_{36} - X_{46} + X_{63} + X_{64} + X_{67} = 0$;
$X_{57} + X_{67} = 1$;
End

```
Global optimal solution found.
  Objective value:                              18.00000
  Infeasibilities:                              0.000000
  Total solver iterations:                             5
  Elapsed runtime seconds:                          0.03

  Model Class:                                        LP

  Total variables:                    17
  Nonlinear variables:                 0
  Integer variables:                   0

  Total constraints:                   8
  Nonlinear constraints:               0

  Total nonzeros:                     51
  Nonlinear nonzeros:                  0


                      Variable           Value        Reduced Cost
                           X12        1.000000            0.000000
                           X13        0.000000            1.000000
                           X14        0.000000            0.000000
                           X23        0.000000            1.000000
```

X25	1.000000	0.000000
X32	0.000000	7.000000
X34	0.000000	7.000000
X35	0.000000	0.000000
X36	0.000000	0.000000
X43	0.000000	3.000000
X46	0.000000	2.000000
X52	0.000000	10.00000
X53	0.000000	4.000000
X57	1.000000	0.000000
X63	0.000000	8.000000
X64	0.000000	14.00000
X67	0.000000	0.000000

Row	Slack or Surplus	Dual Price
1	18.00000	-1.000000
2	0.000000	-9.000000
3	0.000000	-3.000000
4	0.000000	0.000000
5	0.000000	-2.000000
6	0.000000	2.000000
7	0.000000	4.000000
8	0.000000	-9.000000

Ranges in which the basis is unchanged:

Objective Coefficient Ranges:

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
X12	6.000000	1.000000	7.000000
X13	10.00000	INFINITY	1.000000
X14	7.000000	7.000000	2.000000
X23	4.000000	INFINITY	1.000000
X25	5.000000	1.000000	7.000000
X32	4.000000	INFINITY	7.000000
X34	5.000000	INFINITY	7.000000
X35	2.000000	INFINITY	0.000000
X36	4.000000	0.000000	1.000000
X43	5.000000	INFINITY	3.000000
X46	8.000000	INFINITY	2.000000
X52	5.000000	INFINITY	10.00000
X53	2.000000	INFINITY	4.000000
X57	7.000000	1.000000	0.000000
X63	4.000000	INFINITY	8.000000
X64	8.000000	INFINITY	14.00000
X67	5.000000	0.000000	1.000000

Righthand Side Ranges:

Row	Current RHS	Allowable Increase	Allowable Decrease
2	1.000000	0.000000	0.000000
3	0.000000	0.000000	0.000000
4	0.000000	0.000000	0.000000
5	0.000000	0.000000	0.000000
6	0.000000	0.000000	0.000000
7	0.000000	0.000000	0.000000
8	1.000000	0.000000	0.000000