Markov Chain Models in the Study of Covid-19

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GSS/SSS/ASA Practicum on Blended Data

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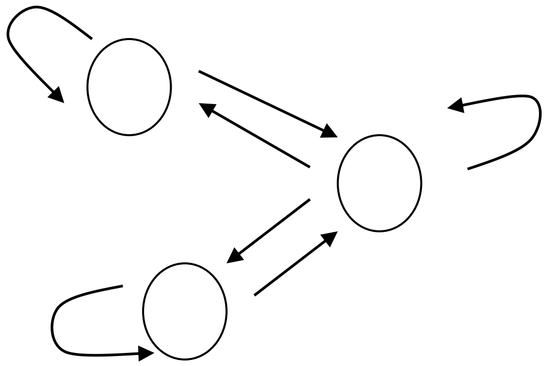
Topics Included in Papers:

- https://www.researchgate.net/publication/343021113_A_Markov_Chain_Model_for_Covid-19_Survival_Analysis
- https://www.researchgate.net/publication/343345908_A_Markov_Model_to_Study_Covid-19_Herd_Immunization
- https://www.researchgate.net/publication/343825461_A_Markov_Model_to_Study_College_Re-opening_Under_Covid-19

Simple Markov Chain over a three-element state space: (0) Non-infected, (1) Infected and (2) Hospitalized population.



Transition Probability Matrix P for Three State *Markov Chain*

$$P = \begin{array}{cccc} States & 0 & 1 & 2 \\ 0 & p_{00} & p_{01} & p_{02} \\ 1 & p_{10} & p_{11} & p_{12} \\ 2 & p_{20} & p_{21} & p_{22} \end{array} = \begin{array}{cccc} States & 0 & 1 & 2 \\ 0 & 1-q_{01} & q_{01} & 0 \\ 1 & p_{10} & 1-p_{10}-q_{12} & q_{12} \\ 2 & 0 & p_{21} & 1-p_{21} \end{array}$$

$$\Pi = Limit_{T\to\infty}(\Pr ob\{X(T)=0\}, \Pr ob\{X(T)=1\}, \Pr ob\{X(T)=2\}) = (\Pi_1;\Pi_2;\Pi_3)$$

Comparisons of two-systems performance measures:

Case (Rates)	Long-run	Not Infected	Infected Home	Hospitalized
Efficient (5%)	Probabilities	0.545	0.273	0.182
Efficient (5%)	Times Between	1.834	3.667	5.50
Inefficient (10%)	Probabilities	0.387	0.322	0.290
Inefficient (10%)	Times Between	2.583	3.099	3.444

Efficient system: *transition probability* (infection) is 0.05 and of remaining in the Hospital (State 3) is 0.7.
Ineficient system: *transition probability* (infection) is 0.1 and of remaining in the Hospital (State 3) is 0.8.

For Efficient System: $T_0 = 1/\pi_0 = 1/0.545 = 1.834$; $T_1 = 1/\pi_1 = 3.667$; $T_2 = 1/\pi_2 = 5.50$;

Markov Chain over five states

Pop;	Infect	Hosp	ICU	Dead	
0.93	0.07	0.00	0.00	0.00	(1); Non-Infected population
0.05	0.80	0.10	0.05	0.00	(2): Infected (isolated at home or hospital)
0.00	0.15	0.80	0.05	0.00	(3): Hospitalized (after becoming ill);
0.00	0.00	0.05	0.80	0.15	(4): In the hospital ICU (or ventilators);
0.00	0.00	0.00	0.00	1.00	(5) Dead (absorbing state; no return)

Matrix inverse $(I-Q)^{-1}$ of Transient States:
Sojourns in Tr. States Before Absortion.

Populat	Infectd	Hospital	ICU
26.1905	16.6667	10.0000	6.66667
11.9048	16.6667	10.0000	6.66667
9.5238	13.3333	13.3333	6.66667
2.3810	3.3333	3.3333	6.66667

Average times to death from all transient states

Starting State for any Individual	**Average Time to Pass Away (Die)**
From Initial Time of Consideration	26.19 + 16.66 + 10.00 + 6.66 = 59.52 days
From the Time of Infection/Isolation	16.67 +10 + 6.67 = 33.34 days
From the Time of Hospitalization	=10.00 + 6.66 = 16.66 days
From the Time of entering ICU/Ventilator	= 6.66 days
Average Time, before becoming infected	26.19 days

Probabilities of a patient dying, starting from any of the transient states

Starting	State	Two Days	Four Days	Eight Days	Sixteen Days
From Initial	**(Healthy)**	0.000	0.002	0.018	0.098
From Time	**of Infection**	0.007	0.036	0.118	0.282
From Time	**Hospitalization**	0.007	0.038	0.127	0.307
From Time	**ICU/Ventilator**	0.270	0.444	0.636	0.780

Probability of a person ever reaching any state from another

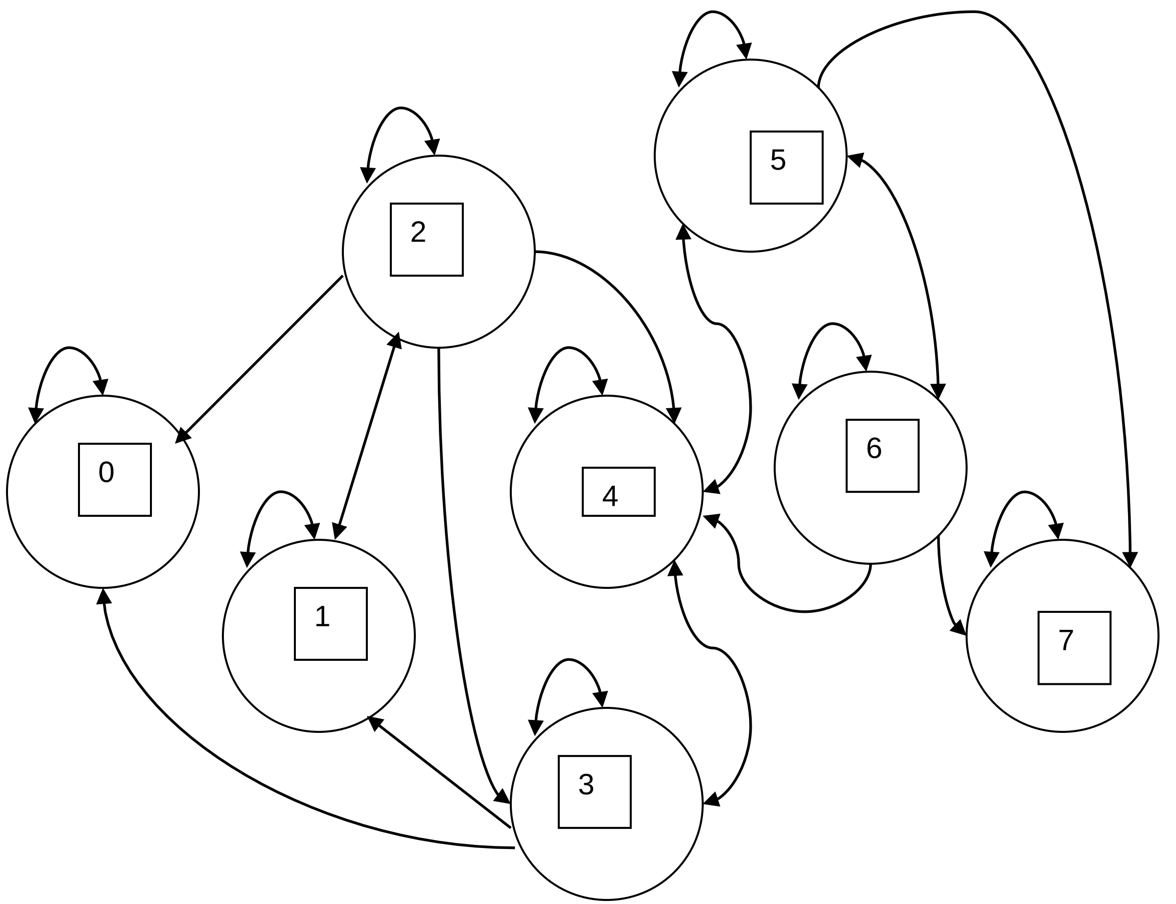
	Healthy	Infected	Hospitalized	ICU/Vent.	Death
Healthy	0.96	1.0	0.75	1.0	1.0
Infected	0.45	0.94	0.75	1.0	1.0
Hospitalized	0.36	0.8	0.92	1.0	1.0
ICU/Vent.	0.09	0.2	0.25	0.85	1.0
Death	0.0	0.0	0.0	0.0	1.0

Markov Chain for Herd Immunization

Over an *eight-element State Space*:

(0) Covid-19 Immunized population
(an absorbing state);
(1) *Non Infected* persons in the General Population;
(2) *Infected persons*
(but asymptomatic; i.e. not known to be such);
(3) *Infected persons Detected and Isolated*;
(after symptoms, or Covid-19 tests positive)
(4) *Hospitalized patients*
(after becoming ill with Colvid-19);
(5) *Patients in the ICU* (very sick);
(6) Patients in a Ventilator (critical);
(7) Patient *Death*
(an absorbing state)

Markov Chain *State Space Diagram*



Markov Chain Transition Probability Matrix

State	Immune	UnInfected	Infected	Isolated	Hospital	ICU	Ventilator	Dead
0	1.0	0.00	0.00	0.0	0.0	0.0	0.0	0.0
1	0.0	0.96	0.04	0.0	0.0	0.0	0.0	0.0
2	0.2	0.05	0.40	0.2	0.2	0.0	0.0	0.0
3	0.2	0.05	0.00	0.6	0.2	0.0	0.0	0.0
4	0.0	0.00	0.00	0.1	0.8	0.1	0.0	0.0
5	0.0	0.00	0.00	0.0	0.3	0.3	0.3	0.1
6	0.0	0.00	0.00	0.0	0.1	0.3	0.3	0.3
7	0.0	0.00	0.00	0.0	0.0	0.0	0.0	1.0

Average times to death from each of the transient states

Starting State for any Individual	**Average Time to Pass Away (Die)**
From Time of Initial Infection (undetected)	=1.667+2.22+5.556+ 0.972+0.417=10.83 days
From the Time of Infection/Isolation	=3.889+5.556+0.972+0.4167= 10.83 days
From the Time of Hospitalization	=11.11+1.94+0.83 = 13.9 days
From the Time of entering an ICU	=2.91667 + 1.25000 = 4.17 days
From the Time of entering a Ventilator	2.08 days

Probability of Dying or Becoming Immune, Starting from a Transient State

Starting State	Probability of Dying	Probability Immunization
Uninfected	0.222222	0.777778
Infected/undetected	0.222222	0.777778
Infected/Isolated	0.222222	0.777778
Hospitalized	0.444444	0.555556
In ICU	0.666667	0.333333
On Ventilator	0.777778	0.222222

Our statistical models are not intended to compete with, but *enrich and complement* the *models developed by Epidemiologists* and Public Health professionals. One Public Health model example:

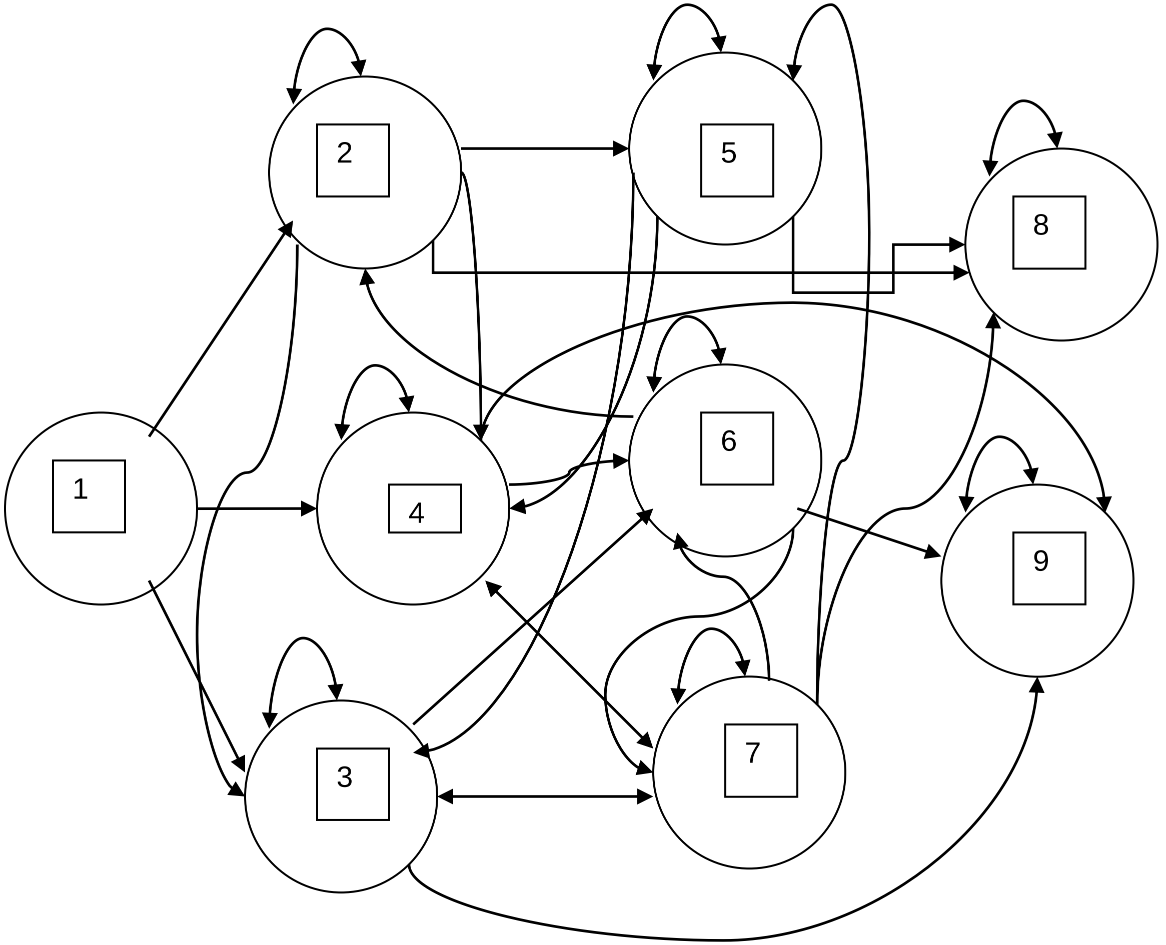
https://www.nature.com/articles/s41591-020-0883-7#Sec2

The total population is partitioned into eight stages of disease: S, susceptible (uninfected); I, infected (asymptomatic or pauci-symptomatic infected, undetected); D, diagnosed (asymptomatic infected, detected); A, ailing (symptomatic infected, undetected); R, recognized (symptomatic infected, detected); T, threatened (infected with life-threatening symptoms, detected); H, healed (recovered); E, extinct (dead).
The interactions among these stages are shown in Fig. 1 of said article. We omit probability rates of becoming susceptible again after having recovered from infection. Although anecdotal cases are found in literature, re-infection rate values appear negligible.

Markov Chain for College Reopening **over a *nine-element State Space***

(1) *Arrival to Campus* and *Covid- 19 testing*;
(2) *Infected students* go *into Isolation* units;
(3) Some students are placed in *Presential* courses;
(4) Other students are placed in *Distance Learning* courses;
(5) Some *students who violated Code* are *placed in Suspension*;
(6) Some *students* become *infected* with Covid-19,
but are *not detected* as such;
(7) Some *students violate* code but are *not detected*;
(8) *Absorption*: Some students are Expelled from College
(9) *Absorption*: Other students Complete their Semester

Markov Chain *State Space Diagram*



Markov Chain Transition Probability Matrix

St.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9.**
1	0	0.05	0.35	0.6	0.00	0.00	0.00	0.00	0.0
2	0	0.70	0.20	0.0	0.05	0.00	0.00	0.05	0.0
3	0	0.00	0.80	0.0	0.00	0.05	0.05	0.00	0.1
4	0	0.00	0.00	0.8	0.00	0.05	0.05	0.00	0.1
5	0	0.00	0.10	0.1	0.70	0.00	0.00	0.10	0.0
6	0	0.50	0.00	0.0	0.00	0.30	0.00	0.00	0.2
7	0	0.00	0.00	0.0	0.70	0.10	0.00	0.20	0.0
8	0	0.00	0.00	0.0	0.00	0.00	0.00	1.00	0.0
9	0	0.00	0.00	0.0	0.00	0.00	0.00	0.00	1.0

Probability of Expulsion or Completion, Starting from a Transient State

Starting State	Probability of Expulsion	Probability of Completion
Arrival	0.216839	0.783161
Infected	0.384030	0.615970
Presential	0.208039	0.791961
Distance Learning	0.208039	0.791961
Suspension	0.472026	0.527974
Infected but undetected	0.274307	0.725693

Conclusions

The *Markov Chain models developed in this paper*, its transition rates, and specific transient and recurrent states, were *built based on our readings about Covid-19* and *our 45+ years of statistics and O.R. experience* in modeling.

Our intent is to place the models in the hands of the specialists, so they can redevelop them with better information.

Models can be improved by those having better information by modifying the state space, the transition rates and/or transition directions, among other modifications.

Alternatively, models can be used as an illustration, if building their own; or to assess and improve public health plans, or by *faculty and students* to assess their risks when participating in such *plans*,

Changing transition rates in the Markov Chain helps study how *they affect absorption probabilities* and thus *compare efficient and inefficient plans*, as well as to *help establish an acceptable community infection rate*