A Markov Model to Study College Re-opening Under Covid-19

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1.0 Introduction

This *Markov Model studies* the problem of *Re-opening Colleges during Covid-19 Pandemia.* We wrote two Markov model papers before: *A Markov Chain Model for Covid-19 Survival Analysis*: https://www.researchgate.net/publication/343021113_A_Markov_Chain_Model_for_Covid-19_Survival_Analysis) We assumed everyone would become infected and die, and in *A Markov Model to Study Covid-19 Herd Immunization* we assumed all survivors would become immune: https://www.researchgate.net/publication/343345908_A_Markov_Model_to_Study_Covid-19_Herd_Immunization?channel=doi&linkId=5f244905458515b729f78487&showFulltext=true

We have also previously written *An Example of Survival Analysis Applied to Covid-19 Data*: https://www.researchgate.net/publication/342583500_An_Example_of_Survival_Analysis_Data_Applied_to_Covid-19 Also *Multivariate Statistics in the Analysis of Covid-19 Data,* and *More on Applying Multivariate Statistics to Covid-19 Data*. Both of these tutorials can be found in: https://www.researchgate.net/publication/341385856_Multivariate_Stats_PC_Discrimination_in_the_Analysis_of_Covid-19 (statistical methods & tools for classification of states, regions, etc.) https://www.researchgate.net/publication/342154667_More_on_Applying_Principal_Components_Discrimination_Analysis_to_Covid-19 according to levels of infection and other metrics, and https://www.researchgate.net/publication/343700072_A_Digression_About_Race_Ethnicity_Class_and_Covid-19 on socio-economic issues affected by Covid-19.

We have also written on the use of *Design of Experiments Applied to the Assessment of Covid-19*, an example of the use of DOE for assessing and controlling the levels of infection in states: https://www.researchgate.net/publication/341532612_Example_of_a_DOE_Application_to_Coronavarius_Data_Analysis We have written *an evaluation of the results of 25 years off-shoring American jobs*, *and its impact on preparedness* to fight the Coronavarus Pandemia, that are in: https://www.researchgate.net/publication/341685776_Off-Shoring_Taxpayers_and_the_Coronavarus_Pandemic as well as on *using methods of reliability data analysis, FTA, and FMEA in the design and operation of ICU* units and Ventilators: https://www.researchgate.net/publication/342449617_Example_of_the_Design_and_Operation_of_an_ICU_using_Reliability_Principles All above-cited papers are in our *ResearchGate* page.

The above work constitutes our *pro-bono collaboration to fight Covid-19*, based on *Proposal*: https://www.researchgate.net/publication/341282217_A_Proposal_for_Fighting_Covid-19_and_its_Economic_Fallout

2.0 A Markov Chain for Re-opening College, with Two Absorbing States.

We analyze the situation of Re-opening College under Covid-19, using a Markov Chain defined over a *nine element state space*. The proposed Markov Chain models a *non-recurrent process* that moves through a number of *Transient states (1 through 7)* eventually *leading to one of two Absorbing States (8 & 9: Expulsion or Coursework Completion)* where students remain forever.

Because the *state space* contains both transient and recurring (absorbing) states, instead of steady state solution we obtain the *long run probabilities of Expulsion or Completion the Course,* when starting in any of the Transient states. Their corresponding expected times are irrelevant here.

Let $\{X_n; n \geq 0\}$ be a *Markov Chain* over a *nine-element State Space* defined as follows:

(1) *Arrival to Campus* and *Covid- 19 testing*;
(2) *Infected students* go *into Isolation* units;
(3) Some students are placed in *Presential* courses;
(4) Other students are placed in *Distance Learning* courses;
(5) Some *students who violated Code* are *placed in Suspension*;
(6) Some *students* become *infected* with Covid-19, but are *not detected*;
(7) Some *students violate* code but are *not detected*;
(8) *Absorption*: Some students are Expelled from College
(9) *Absorption*: Other students Complete their Semester

The *Transition Probability Matrix P* for this *Markov Chain* is:

St.	1	2	3	4	5	6	7	8	9.
1	0	0.05	0.35	0.6	0.00	0.00	0.00	0.00	0.0
2	0	0.70	0.20	0.0	0.05	0.00	0.00	0.05	0.0
3	0	0.00	0.80	0.0	0.00	0.05	0.05	0.00	0.1
4	0	0.00	0.00	0.8	0.00	0.05	0.05	0.00	0.1
5	0	0.00	0.10	0.1	0.70	0.00	0.00	0.10	0.0
6	0	0.50	0.00	0.0	0.00	0.30	0.00	0.00	0.2
7	0	0.00	0.00	0.0	0.70	0.10	0.00	0.20	0.0
8	0	0.00	0.00	0.0	0.00	0.00	0.00	1.00	0.0
9	0	0.00	0.00	0.0	0.00	0.00	0.00	0.00	1.0

The corresponding *state space diagram* is below:

Figure 1: The *State Space Diagram* for this Markov Chain is:
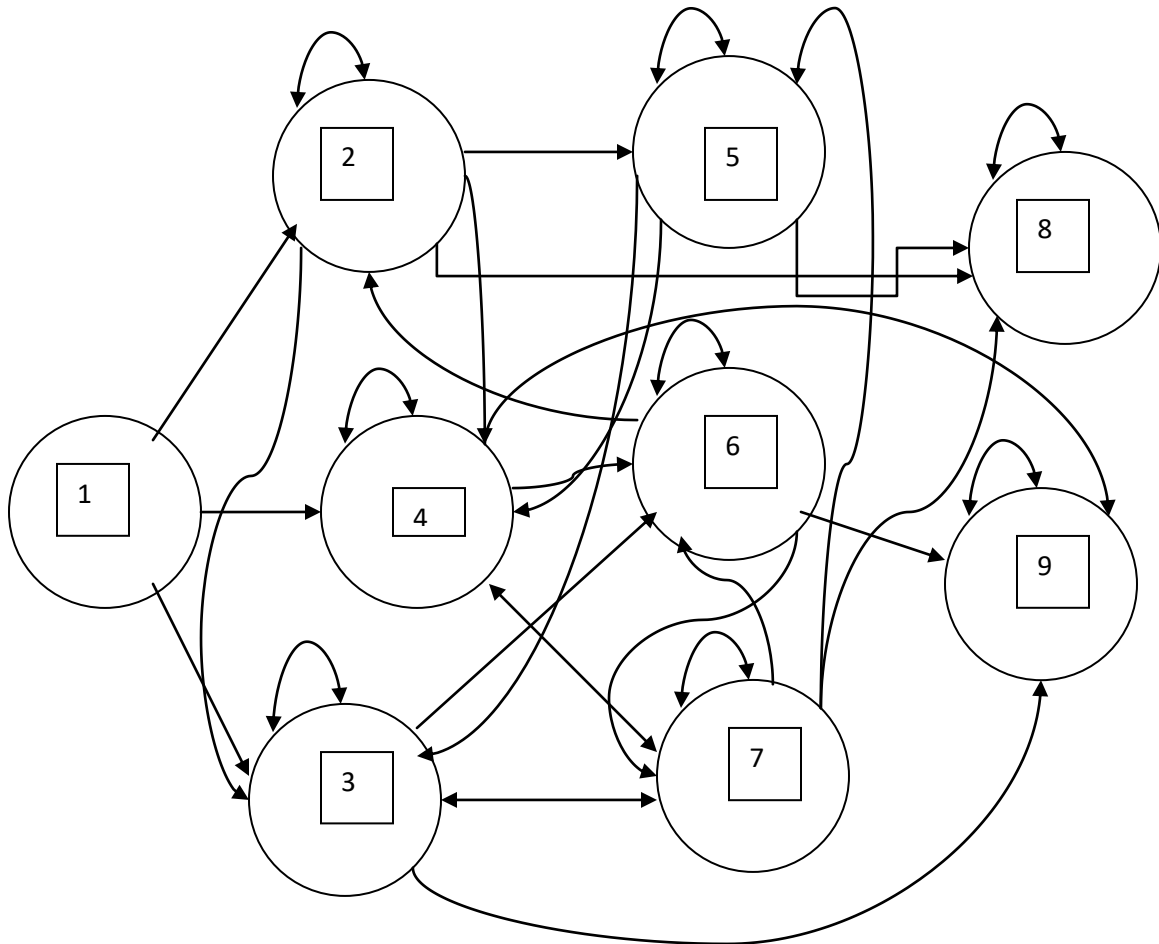
The *Markov Chain unit time* is *a day*. *Transitions* refer to the *State changes* that occur from *one morning to the following morning*. Students arrive (1), are tested and distributed into isolation (2) if infected (5%), Presential (3), and Distance courses (4). Some students become infected (6) and are again Isolated (if detected) or remain, asymptomatic, until the end of the semester (possibly spreading the disease). Some students violate Code by partying, gathering, etc. (7), and some get caught and are Suspended (5) or become infected (6), while others are not detected and return to their courses. Some isolated students break their status, are caught and are Suspended (5). Some Code violations are so blatant that students caught are Expelled (8), as also are some students that break their Suspension. Students Expelled do not return to Campus. All other students finish their semester (9) after completing Isolation, Suspension, and Distance or Presential coursework.

The transition rates depend on environmental conditions (e.g. infection rates, social distancing, mask compliance), on modeler's experience and concomitant information. Specific transitions between states also stem from modeler's experience, information and judgment call. All such adjustments constitute the actual *tuning up* of the present Markov model, when used by others.

We obtain, from P, the Sub-Matrix Q, corresponding to the seven Transient states:

Matrix Q of the Seven Transient States: Matrix CollegeQ

1	2	3	4	5	6	7.
0	0.05	0.35	0.6	0.00	0.00	0.00
0	0.70	0.20	0.0	0.05	0.00	0.00
0	0.00	0.80	0.0	0.00	0.05	0.05
0	0.00	0.00	0.8	0.00	0.05	0.05
0	0.00	0.10	0.1	0.70	0.00	0.00
0	0.50	0.00	0.0	0.00	0.30	0.00
0	0.00	0.00	0.0	0.70	0.10	0.00

We then subtract matrix Q from the Identity Matrix, yielding (I-Q),

Matrix I − Q: Matrix IminCollQ

1	2	3	4	5	6	7.
1	-0.05	-0.35	-0.6	0.00	0.00	0.00
0	0.30	-0.20	0.0	-0.05	0.00	0.00
0	0.00	0.20	0.0	0.00	-0.05	-0.05
0	0.00	0.00	0.2	0.00	-0.05	-0.05
0	0.00	-0.10	-0.1	0.30	0.00	0.00
0	-0.50	0.00	0.0	0.00	0.70	0.00
0	0.00	0.00	0.0	-0.70	-0.10	1.00

We invert the above: **Matrix inverse S = (I-Q)$^{-1}$**

The Potential Matrix (of which S is a sub-matrix) contains the *Long-Run Sojourns* (the average number of visits) *to each of the States* (matrix columns), when starting from any other of the States (represented by the matrix rows). The Sojourns that occur outside Sub-matrix S are either Zero (if the states cannot be reached) or Infinite (if it is to an Absorbing state).

Inverse S = (I-Q)$^{-1}$: Matrix InvIminCollQ

1	2	3	4	5	6	7.
1	1.05296	3.28551	3.48255	0.96510	0.53178	0.33840
0	4.03042	4.67681	0.64639	1.29278	0.41825	0.26616
0	0.89625	6.37018	0.47393	0.94785	0.53775	0.34221
0	0.89625	1.37018	5.47393	0.94785	0.53775	0.34221
0	0.59750	2.58012	1.98262	3.96524	0.35850	0.22814
0	2.87887	3.34058	0.46171	0.92341	1.72732	0.19011
0	0.70614	2.14014	1.43400	2.86801	0.42368	1.17871

State Sojourns and times to absorption, in the context of the present problem, are of small value, as the *coursework time is pre-determined* (16 weeks) and students either complete it or not.

3.0 Results for the Two (*Expulsion or Coursework Completion*) Absorbing States

As explained above, state sojourn times are nor relevant, in this case.

We calculate instead the *Probabilities of Expulsion and Coursework Completion, starting from any of the transient states*. We calculate these probabilities *using Matrix B*, a sub-matrix of all transient states. B is constructed by taking the rows and columns of P, above the Identity sub-matrix corresponding to the two Absorbing states (last two rows).

Matrix B: Matrix CollegeB

```
0.00   0.0
0.05   0.0
0.00   0.1
0.00   0.1
0.10   0.0
0.00   0.2
0.20   0.0
```

Using Matrix S = (I-Q)$^{-1}$ we calculate Matrix G = S*B:

Such Matrix G, yields the Probability Of Ever Reaching

Absorbing states Expulsion, or Coursework Completion:

Starting from any given transient State:

St.	Expulsion	CourseEnd
1	0.216839	0.783161
2	0.384030	0.615970
3	0.208039	0.791961
4	0.208039	0.791961
5	0.472026	0.527974
6	0.274307	0.725693
7	0.557849	0.442151

Matrix G provides the probabilities of ever reaching one of the two Absorbing States (Expulsion or Completion) starting from any of the Transient states.

Table #1 Probability of Expulsion or Completion, starting from a Transient State

Starting State	Probability of Expulsion	Probability of Completion
Arrival	0.216839	0.783161
Infected	0.384030	0.615970
Presential	0.208039	0.791961
Distance Learning	0.208039	0.791961
Suspension	0.472026	0.527974
Infected but undetected	0.274307	0.725693

Notice how the *Probability of Expulsion*, the day of student first arrival to Campus, is 0.217 (and that of completing the course is, of course, 0.783). However, the Probability of Expulsion, if the student is detected as infected, and then isolated, is 0.38. The *Probability of Expulsion,* if placed in Suspension, is 0.47. Increase may occur due to having more opportunities to run into problems with the administration (e.g. breaking Isolation or Suspension) leading into student Expulsion.

On the other hand the *Probability of Course Completion is, from most states, close to 0.8, except for students that have been infected or suspended.* In particular, the *Probability of Completing the Course, for students* that have been *infected* with Covid-19, but have *not been detected* as such, is 0.62. This means that such may be participating in college activities, possibly infecting inadvertently others. This result underlines *the need of retesting* for Covid-19, multiple times.

Transition (infection) rates depend on student behavior and on college prevention plans. They *can be changed* accordingly, *to assess best, worst and average cases*, and take decisions. Such modifications of the models are used in what *Operations Researchers call the What If Game* (e.g. what would happen if infection rate were 9% instead of 4%) to evaluate alternatives.

4.0 Discussion

This Markov model, due to its specific State Spaces E_j and its transition probabilities (X_{ij} for i,j = 0, … , n) is particularly *useful to assess the effectiveness of reopening plans*.

Different *infection (transition) rates* help study their *impact on the probabilities of Expulsion and Course Completion*. Infection rates depend on student compliance with all community and public health measures, such as face covering, social distancing, etc. By assigning different values to the infection (transition) rates, *their impact* on the Re-opening plans can be *assessed and compared*.

Transition rates and number of States depend on student behavior (which in turn is *impacted by community spread and public health measures* such as face covering, social distancing, etc.) as well as on *specifics of the reopening plans*. By using different transition rates, the resulting *Reopening plan Logistics problems* can be *assessed and addressed*.

Probabilities of Expulsion and Completion, starting from different States E_j, can be *combined with student registration information* to estimate the number students completing coursework that result from different scenarios and plans, thus helping select the most efficient one.

Statistics and mathematical models are tools that help *answer practical questions*, as well as to *compare the performance* of different alternatives, *in a more objective, non-partisan* way.

Further readings about Markov Processes can be found in the Bibliography Section.

5.0 Conclusions

The *Markov Chain model developed in this paper*, its transition rates, and specific transient and recurrent states, were *built based on our readings about Covid-19* and *our 45+ years of statistics and O.R. experience* in modeling and data analysis, working for several research and academic organizations. This author is not a public health specialist, but a statistician. *Our intent is to place the model in the hands of the specialists*, so they can redevelop it with better information.

This model can be improved by those having better information by modifying the state space, the transition rates and/or transition directions, among other modifications. Alternatively, it can be used as an illustration, if building their own. Once updated and *fine tuned, or rebuilt* differently, the *Markov model can be used by college* authorities to assess and improve their reopening plans, by *faculty and students* to assess their risks when participating in such re-openings, *and by the government* authorities *to assess the validity and safety of such plans*, to allow or proscribe them.

Results from the Markov Chain model, most useful for Covid-19 problems, have to do with the *probabilities of absorption* (Expulsion and Completion) and are given in *Section 3.0*.

Changing the transition rates in the Markov Chain (*Section 2.0*) helps study how *they affect the absorption probabilities* (Expulsion and Completion) starting from each state, and thus *compare efficient and inefficient plans*. This may also *help establish an acceptable community infection rate* under which such College Reopening operation presents a lower risk (and vice-versa).

Finally, *our statistical models* are not intended to compete with, but to *complement and enrich*, the *models developed by Epidemiologists* and other Public Health professionals.

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Jorge Luis Romeu retired Emeritus from the State University of New York (SUNY). He was then, for sixteen years, a Research Professor at Syracuse University, where he is currently an Adjunct Professor of Statistics. Romeu worked for many years as a Senior Research Engineer at the Reliability Analysis Center (RAC), an Air Force Information and Analysis Center operated by IIT Research Institute (IITRI). Romeu received seven Fulbright assignments: in Mexico (3), the Dominican Republic (2), Ecuador, and Colombia. He holds a doctorate in Statistics/O.R., is a C. Stat Fellow of the Royal Statistical Society (RSS), a Member of the American Statistical Association (ASA), and of the American Society for Quality (ASQ). Romeu is a Past ASQ Regional Director, and holds Reliability and Quality ASQ Certifications. He created and directs the *Juarez Lincoln Marti Int'l. Education Project* (https://web.cortland.edu/matresearch/) which supports higher education in Ibero-America. Readers can find all our work on Coronavarus (as well as on many other subjects) in our *Quality, Reliability and Continuous Improvement Institute* (QR&CII) web page:

https://web.cortland.edu/romeu/QR&CII.htm