LP SENSITIVITY ANALYSIS

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SENSITIVITY ANALYSIS

Defined:

- A method of discovering how the optimal solution is altered by changes, within certain ranges of the objective function coefficients and the right-hand side values

Implemented:

- By mangers who work in a dynamic setting with inexact estimates of the coefficients
- Also assists managers to ask particular what-if-questions about the problem

GRAPHICAL SENSITIVITY ANALYSIS

- Graphical solution methods can be used to perform sensitivity analysis on the objective function coefficients and the right–hand-side values for the constraints for Linear Programming problems with two decision variables

EXAMPLE 3 PAGE 124

Min 8X+12Y

s.t.

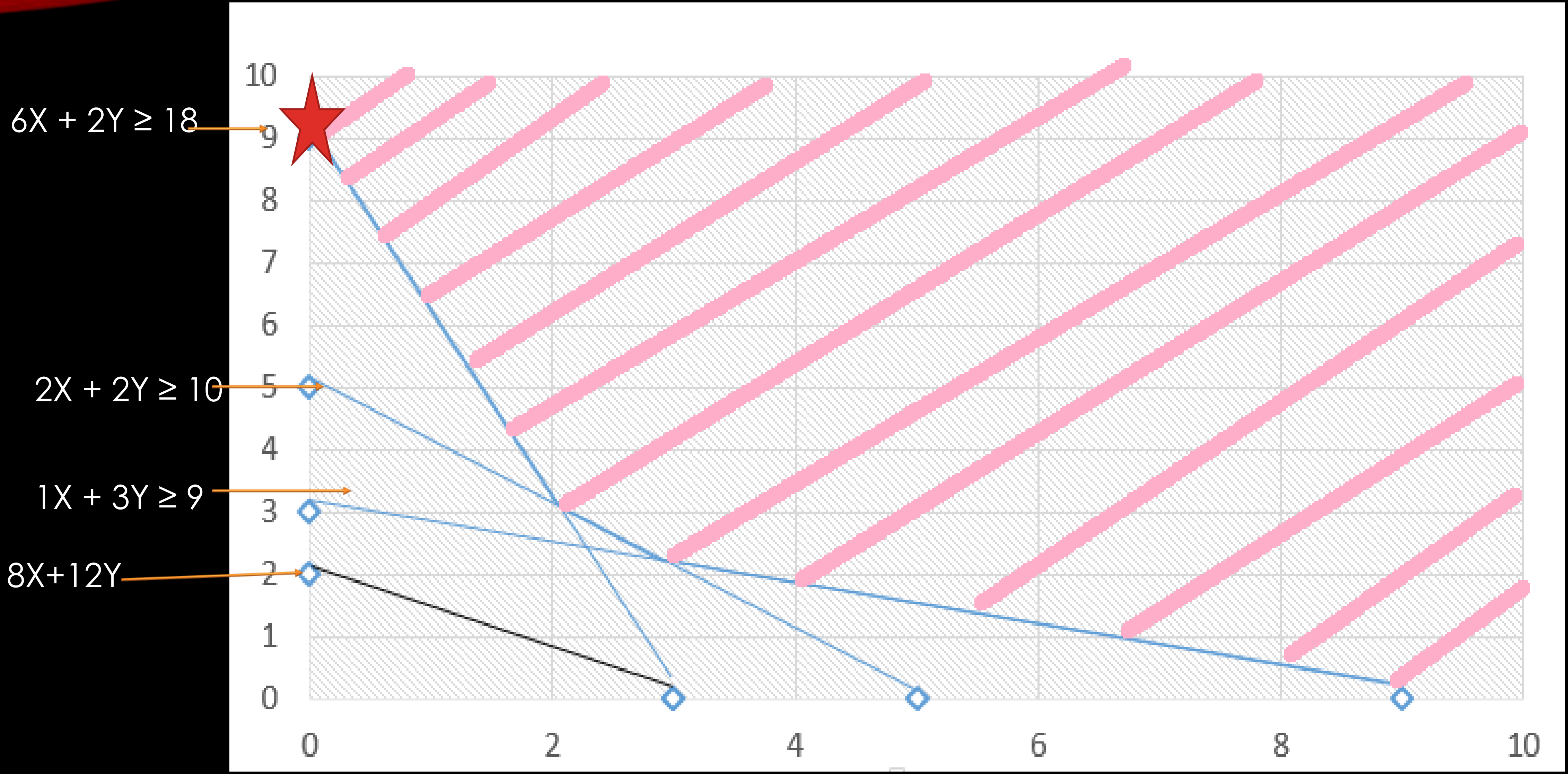
$1X + 3Y \geq 9$

$2X + 2Y \geq 10$

$6X + 2Y \geq 18$

$A, B \geq 0$

GRAPHICAL SOLUTION OF PAGE 124 QUESTION 3



SENSITIVITY ANALYSIS ON SOLVER

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B5	Solution x	0	-16	0	0	0
C5	Solution y	9	-16	0	0	0

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
A10	Constraint	27	-8	0	0	0
A11	Constraint	18	-4	0	0	0
A12	Constraint	18	-4	0	0	0

EXAMPLE 12 PAGE 130

Max 63E + 95S + 135D

s.t.

IE + IS + ID ≤ 200

IE + 2S + 4D ≤ 320

8E + 12S + 14D ≤ 2400

E, S, D ≥ 0

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$13	Solution: Economy	80	0	63	12	15.5
\$C\$13	Solution: Standard	120	0	95	31	8
\$D\$13	Solution: Deluxe	0	-24	135	24	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$A\$18	Constraints	200	31	200	80	40
\$A\$19	Constraints	320	32	320	80	120
\$A\$20	Constraints	2080	0	2400	1E+30	320

RANGE OF OPTIMALITY

Supplies the range of values that will allow the current solution to continue to be optimal

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B13	Solution: Economy	80	0	70	5	22.5
C13	Solution: Standard	120	0	95	45	3.333333333
D13	Solution: Deluxe	0	-10	135	10	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
A18	Constraints	200	45	200	80	40
A19	Constraints	320	25	320	80	120
A20	Constraints	2080	0	2400	1E+30	320

RANGE OF OPTIMALITY

Variable Cells

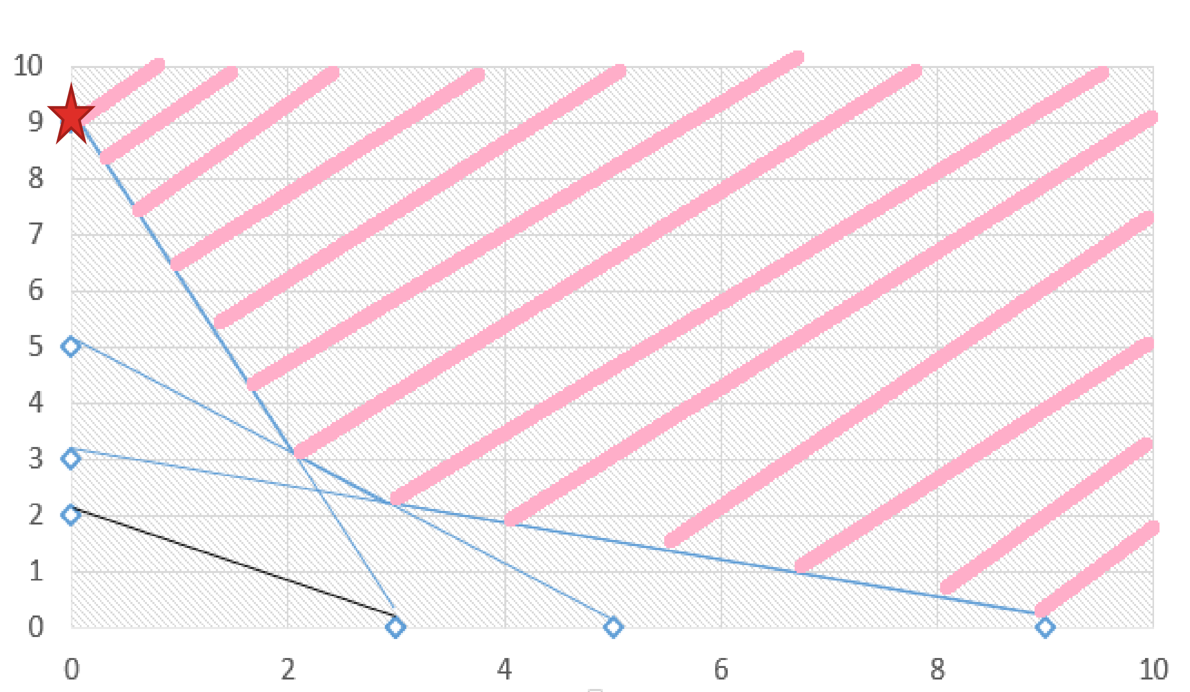
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B13	Solution: Economy	160	0	70	65	10
C13	Solution: Standard	0	-6.666666667	85	6.666666667	1E+30
D13	Solution: Deluxe	40	0	135	145	20

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
A18	Constraints	200	48.33333333	200	93.33333333	120
A19	Constraints	320	21.66666667	320	280	120
A20	Constraints	1840	0	2400	1E+30	560

GRAPHICAL SOLUTION OF PAGE 124 QUESTION 3

Graphically, the limits of a range of optimality are found by changing the slope of the objective function line within the limits of the slopes of the Binding constraint lines.



RIGHT-HAND SIDES

- A change in the right hand side for a constraint may affect the feasible region and perhaps cause a change in the optimal solution.

- As the right-hand side increases , other constraints will become binding and limit the change in the value of the objective function

- **Dual Value** - The change in the value of the optimal solution per unit increase in the right-hand side

DUAL VALUE

- Graphically, a dual value is determined by adding one to the right hand side value and then resolving for the optimal solution in terms of the same two binding constraints.
- The dual value is equal to the difference in the value of the objective functions between the new and original problems.
- The dual value for a nonbinding constraint is **0**.
- A **negative** dual value indicates that the objective function will not improve if the right hand side is increased.

Shadow Price	Constraint R.H. Side
31	200
32	320
0	2400

RANGE OF FEASIBILITY

- Defined: For a change in the right hand side value is the range of values for this coefficient in which the original dual value remains constant.
- Graphically, the range of feasibility is determined by finding the values of a right hand side coefficient such that the same two lines that determined the original optimal solution continue to determine the optimal solution for the problem.

RANGE OF FEASIBILITY

The range over which the dual value is applicable

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B13	Solution: Economy	160	0	63	12	15.5
C13	Solution: Standard	80	0	95	31	8
D13	Solution: Deluxe	0	-24	135	24	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
A18	Constraints	240	31	240	40	80
A19	Constraints	320	32	320	40	80
A20	Constraints	2240	0	2400	1E+30	160

RANGE OF FEASIBILITY

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B13	Solution: Economy	284.4444444	0	63	14.14285714	4.5
C13	Solution: Standard	0	-5	95	5	1E+30
D13	Solution: Deluxe	8.888888889	0	135	117	22.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
A18	Constraints	293.3333333	0	300	1E+30	6.666666667
A19	Constraints	320	11	320	365.7142857	20
A20	Constraints	2400	6.5	2400	40	1280

REDUCED COST

- The reduced cost associated with a variable is equal to the dual value of the non-negativity constrain associated with the variable.
- In general, if a variable has a non-zero value in the optimal solution, then it will have a reduced cost equal to 0.

LIMITATIONS OF CLASSICAL SENSITIVITY ANALYSIS

- **Simultaneous Changes** - The range analysis for objective function coefficients and the constraint right-hand sides is only applicable for changes in a single coefficient.
- **Changes in Constraint Coefficients** – Classical sensitivity analysis provides no information about changes resulting from a change in a coefficient of a variable in a constraint.
- **Non-intuitive Dual Values** – Constraints with variables naturally on both the left-hand and right-hand sides often lead to dual values that have a non-intuitive explanation. This is often the case with constraints that involve percentages.

EXAMPLE 12 ON LINGO

Solution:

Global optimal solution found.

Objective value:	16440.00
Infeasibilities:	0.000000
Total solver iterations:	4
Model Class:	LP
Total variables:	3
Nonlinear variables:	0
Integer variables:	0
Total constraints:	4
Nonlinear constraints:	0
Total nonzeros:	12
Nonlinear nonzeros:	0

Variable	Value	Reduced Cost
E	80.00000	0.000000
S	120.0000	0.000000
D	0.000000	24.00000

Row	Slack or Surplus	Dual Price
1	16440.00	1.000000
2	0.000000	31.00000
3	0.000000	32.00000
4	320.0000	0.000000