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A SMALL SAMPLE MONTE CARLO STUDY OF FOUR SYSTEM RELIABILITY BOUNDS

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Abstract—Four theoretical methods providing approximations to total system reliability bounds from subsystem test data are compared through a Monte Carlo study. The system bounds in question, Kraemer, El Mawazini-Buehler, Grubbs and Mann–Grubbs, are valid asymptotically, i.e. when the number of failures observed is large. However, their small sample properties, and the closed forms of their small sample distributions, are unknown. In practice, when a system cannot be tested as a whole, the number of observed failures may be small. Hence, the practical importance for the study of the small sample properties of these bounds. Lacking this information, we perform a numerical study of these small sample properties, as well as a comparison of the bounds.

1. INTRODUCTION

In reliability studies sometimes it is not possible to test a system as a whole, but by subcomponent units. This may occur, for example, when operating the entire system would imply its destruction or loss. However, it is still necessary to estimate the reliability of the total system in question.

Extensive research in this area has been performed. As a result, several theoretical [1] and empirical [2] system bounds for assessing the reliability of the entire system from subsystem test data have been proposed (a reliability bound $f(\mathbf{X})$, is a one sided confidence interval calculated from component test data $\mathbf{X}$ so that the true system reliability exceeds $f(\mathbf{X})$ with probability $(1-\alpha)$. Hence, the practical problem of comparing these procedures and selecting the "best" bound among those available.

In the present study we consider four well known theoretical bound approximations; Kraemer [3], El Mawazini–Buehler [4], Grubbs [5] and Mann–Grubbs [6] (Table 1). The four bound approximations are applied to series systems composed of 10 subsystems with exponential lifetimes, whose failure rates are estimated from Type II censored subsystem data.

If the (small sample) exact distribution of these system bounds were available, such a comparison is performed using their moments and relative efficiencies. We cannot, however, invoke the asymptotical properties in the presence of just a few failures. In the studies previously conducted to compare these bounds (e.g. [1], [5] and [6]) the authors calculate only a reduced number of examples from very simple series systems and compare their bound approximations with the exact result (El Mawazini [7]). This is due to both the complexity and the round-off errors in the calculations of these quantities for more complex systems.

It is still necessary to know what each bound's expected coverage (of the true system reliability) will be. We still want to know what each bound's variance, skewness and kurtosis will be, for the small sample statistic will not be symmetrically distributed. We will want to know how these bounds will be affected by several factors inherent in reliability studies, e.g. early termination, different and decreasing subsystem reliabilities. According to these properties and their robustness to the factors mentioned above, we want to rank these bound approximations.

Figures 1 through 4 illustrate this situation for our system of 10 subsystems with total reliability or 0.9511. The empirical pdf's (histograms) of the four bound approximations shown, were obtained by simulating 1000 (small sample) system tests and, then, calculating each of the four bounds from them. Notice how, in addition to coverage, these bounds differ in other statistical (performance) measures like median, mode, variance, skewness and kurtosis. Our comparison of these bounds will be based upon these measures.

Table 1. Functional forms of the bound approximations

(i) *Kraemer*

$$f_1(\mathbf{X}) = \exp\left\{\frac{\chi^2_{2r,\alpha}\cdot t_m}{2\cdot\chi_{(1)}}\right\}$$

(ii) *El Mawazini–Buehler*

$$f_2(\mathbf{X}) = \exp\left\{-t_m\left(\sum_j^k \frac{r_j - 1}{X_j} + \xi_\alpha\left[\sum_j^k \frac{r_j - 1}{X_j^2}\right]^{1/2}\right)\right\}$$

(iii) *Grubbs*

$$f_3(\mathbf{X}) = \exp\left\{-m\cdot t_m\left[1 - \frac{2}{9\gamma} - \xi_\alpha\sqrt{\frac{2}{9\gamma}}\right]^3\right\}$$

$$m = \sum_j^k \frac{r_j}{X_j};\quad v = \sum_j^k \frac{r_j}{X_j^2};\quad \gamma = \frac{2m^2}{v}$$

(iv) *Mann–Grubbs*

$$f_4(\mathbf{X}) = \exp\left\{-t_m\cdot m\left[1 - \frac{v}{9m^2} + \xi_\alpha\frac{v}{3m}\right]^3\right\}$$

$$m = \sum_j^k \frac{r_j - 1}{X_j} + X_{(1)}^{-1};\quad v = \sum_j^k \frac{r_j - 1}{X_j^2} + X_{(1)}^{-2}$$

Where:
X_i is the total time on test for the ith subsystem.
K is the total number of subsystems in the system.
$X_{(1)} = \min(X_1, \ldots, X_K)$
r_j = number of failures in subsystem $j = 1, \ldots, K$.
$\mathbf{X} = (X_1, \ldots, X_K)$.
ξ_α = upper α-percentile, standard normal.

The present Monte Carlo study attempts to do just that, numerically. We define a "representative" system with known total and subsystem reliabilities and a set of "representative" testing conditions. We then simulate 1000 tests for each set of "representative" conditions. From these tests we calculate the four reliability bound approximations in question and then perform statistical analyses on these experimentally obtained bounds, as a surrogate procedure for the theoretical small sample pdf's that we do not have nor can obtain.

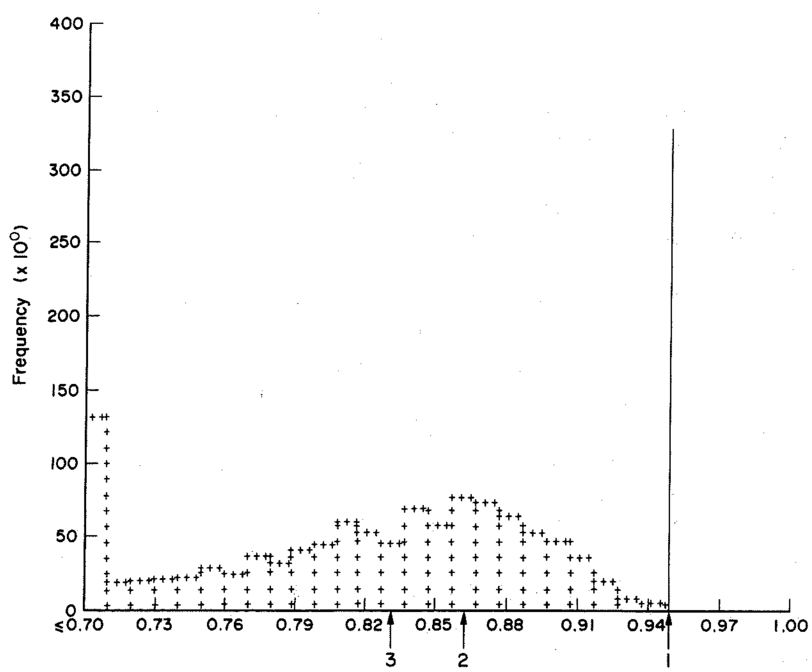

Fig. 1. Kraemer bound for confidence level of 0.75. 1, Real system reliability (0.951). 2, Empirical mode (0.865). 3, Empirical median (0.832). 4, Empirical coverage (1.00). 5, Empirical interquartile range (0.102).

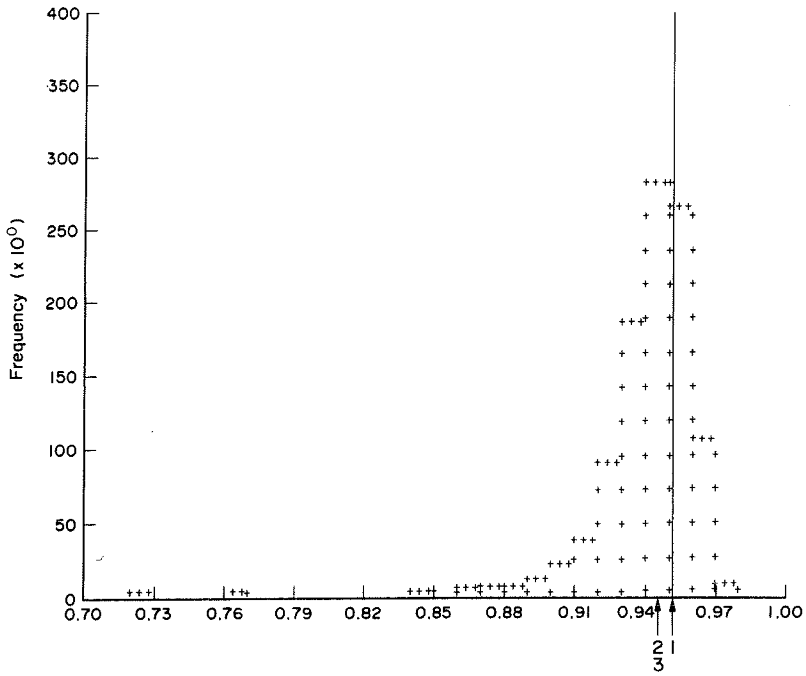

Fig. 2. El Mawazini–Buehler bound for confidence 0.75. 1, Real system reliability (0.951). 2, Empirical mode (0.945). 3, Empirical median (0.945). 4, Empirical coverage (0.661). 5, Empirical interquartile range (0.020).

2. THE SIMULATOR

The simulation model for this study mimics a "representative" highly reliable system which cannot be tested altogether, but only at the subsystem level. Statistical assumptions for the validity of the four reliability bounds under study are given in Table 2 and are met by our simulated system.

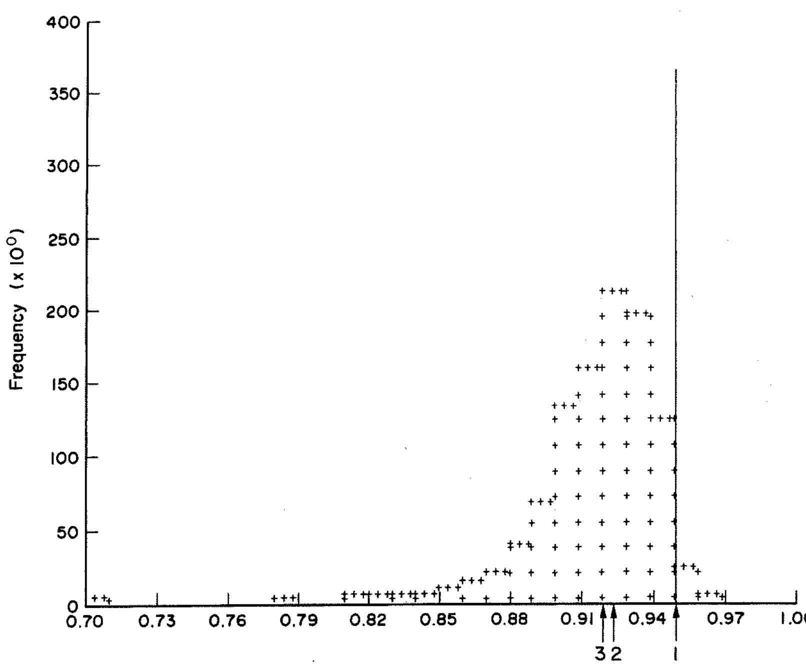

Fig. 3. Grubbs bound for coefficient level of 0.75. 1, Real system reliability (0.951). 2, Empirical mode (0.925). 3, Empirical median (0.922). 4, Empirical coverage (0.982). 5, Empirical interquartile range (0.027).

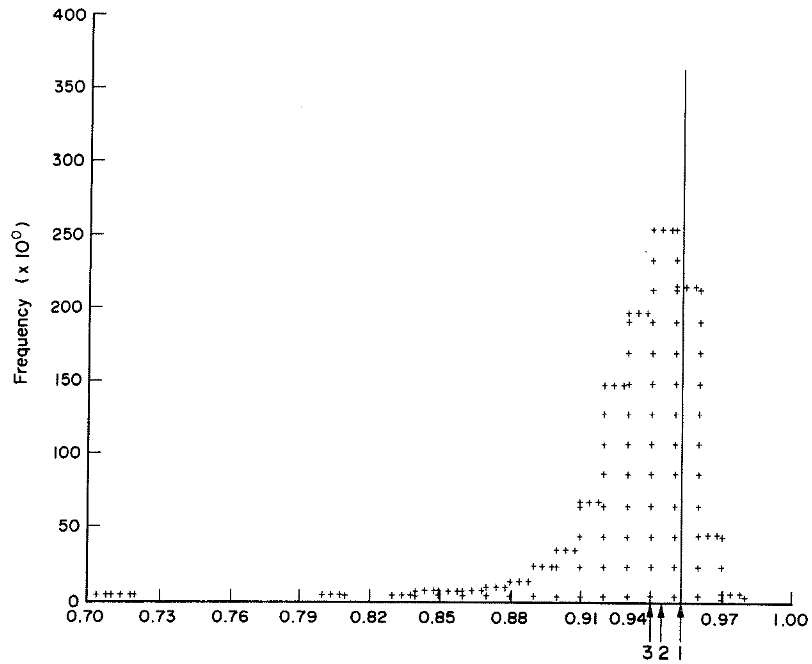

Fig. 4. Mann–Grubbs bound for confidence level 0.75. 1, Real system reliability (0.951). 2, Empirical mode (0.945). 3, Empirical median (0.940). 4, Empirical coverage (0.775). 5, Empirical interquartile range (0.022).

Our "representative" system is composed of 10 independent subsystems with pre-specified reliabilities (Table 3). Because it is a series system, total system reliability is immediately known (their product). There are 15 items of each subsystem placed simultaneously on test. A maximum of three failures will be observed in each subsystem. Mission time (t_m) for the system and all subsystems will be the same. The data is standardized by dividing each failure time by t_m.

Confidence levels for all system bounds are 0.5, 0.75 and 0.90. Pseudo random subsystem failure times were generated by inversion of the exponential distributions (Table 3) using the IMSL library's GGUBS random number generator routine on a Honeywell 6000 computer running on GCOS operating system.

Table 2. Basic statistical assumptions

—Series system
—Independent components or subsystems
—Exponentially distributed Failure Times
—At least one observed Failure in each subsystem (ideally, many more than one)
—All subsystems test are fixed sample and Type II censored
—Exact failure times of all failed items are known
—Results are conditional (fiducial) on observed total times on test (Approximations in [4], [5] and [6] are obtained by inverting the original Reliability distributions and then expanding in Taylor series. To invert the distributions, authors condition on the observed total times on test—as if these were parameters of the distribution).

Table 3. Values for the 10 cases of the simulation of system reliability

Case	Subsystems		Total system reliability
(1)	$(0.995)^{10} \times 1$	=	0.9511
(2)	$(0.995)^{9} \times 0.99$	=	0.9463
(3)	$(0.995)^{9} \times 0.98$	=	0.9367
(4)	$(0.995)^{9} \times 0.97$	=	0.9272
(5)	$(0.995)^{9} \times 0.96$	=	0.9177
(6)	$(0.990)^{10} \times 1$	=	0.9044
(7)	$(0.990)^{9} \times 0.97$	=	0.8861
(8)	$(0.990)^{9} \times 0.95$	=	0.8678
(9)	$(0.990)^{9} \times 0.93$	=	0.8496
(10)	$(0.990)^{9} \times 0.90$	=	0.8222

We defined two testing cases. Case one tests every subsystem until the third failure occurs and then calculates the bounds based on the three first failures of each subsystem. Case two follows the same procedure except that it stops testing the subsystem with the longest time-to-first-failure at its first failure.

The rationale for defining such a system and testing procedures was based on our experience of several years of work with the Reliability Analysis Center (RAC/RADC). Systems with more than five subsystems had not been previously compared, due to complexity and round-off errors, so ten subsystems seemed a reasonable/feasible number. The minimum number of failures required for application of the procedures is one. Hence three failures is reasonable for the "small" sample case.

The validity for independent and series subsystems is based upon the small interdependence in the operation of each subsystem on the others, but the absolute necessity of every component to be working for the entire system to be in operating conditions. The reliability levels defined for the subsystems point towards highly reliable systems: to investigate the effects of a "bad apple" among the subsystems, and the capacity of detection of such a situation by the respective procedures. Exponentiality of failure times is traditionally accepted in electronic equipment. Early termination of the test with the longest 1st failure is a real life constraint in reliability studies.

Each testing setting was defined as a triplet (reliability, confidence level, testing scheme). Each was taken from (i) one (of ten possible) subsystem reliability combinations, (ii) one (of three) bound confidence level and (iii) one of the testing schemes (with/without early termination). For each of these ($10 \times 3 \times 2 = 60$) experimental settings, 1000 "testing experiments" were simulated and each of the four system bound approximations, calculated. The number of replications (1000) was assumed large enough to provide smooth histograms (to approximate the corresponding pdf's) and efficient estimators of the (real) distribution parameters: mean, median, mode, variance, skewness and kurtosis. These parameters were used to characterize and to compare each bound distribution. We restate that the motivation for this whole numerical study is the lack of the exact distribution for the small sample case and the search for a surrogate of this distribution, for study and comparison purposes. For more details the reader is referred to the original document [8].

3. RELIABILITY BOUND APPROXIMATION PERFORMANCES

Throughout this experiment we were investigating four very specific problems (or factors) that could affect our system bound approximations. They were:

(i) The effects of decreasing system reliability, specifically through the presence of a "bad apple" subsystem.

(ii) The effect of changing the system structure (two such reliability structures are defined in Table 3: cases 1–5 and cases 6–10). Any empirical study necessarily has to be performed on a specific system configuration (ours is a series system of 10 components). We wanted to assess how a change in the structure—i.e. another distribution of reliabilities among the subsystems—in such a configuration would affect the experimental results.

(iii) The effects of increasing the confidence $(1-\alpha)$ level of our system bounds (i.e. 0.50, 0.75 and 0.90).

(iv) The effects of changing the testing scheme (specifically, the effect of early truncation).

In order to numerically study the effect of these four factors on our bounds we defined the following statistical location and dispersion performance measures:

(1) With respect to location:

(a) Coverage of the true value of the (reliability) parameter, to assess accuracy through the comparison with nominal coverage.

(b) Median and mode of the 1000 "experiments" (sample) obtained for confidence coefficient of 0.5. We wanted to investigate where the bounds clustered.

(c) Worst case analysis: the largest value (of the 1000 "experimental" bounds) can say something about *how bad* we could do. Since all experimental bounds were obtained in the same settings and the settings all had 1000 cases, the distribution of this order statistic would be the same for all our bounds under the hypothesis of no difference (H_0). This would allow, in our case, comparison across bounds.

(2) With respect to dispersion:

(a) Coefficient of variation (CV), standardized measure of the (parametric) variance of the bounds. It is not very informative in highly skewed distributions (as in our case).

(b) Interquartile range, is a (nonparametric) alternative to measure variation as well as skewness (in combination with the median) and *kurtosis* (in combination with the mode). We wanted to investigate, in addition to the percent of coverage, also the distribution of this coverage about the true value (through the form of the pdf, i.e. skewness and kurtosis).

A statistical and graphical analysis of the above performance measures follows.

4. ANALYSIS OF THE RESULTS

Table 4 compares the bound coverage as system reliability increases from 0.822 to 0.951. The closest to the nominal confidence coefficient $(1-\alpha)$, the better. We can see how Mann–Grubbs is the "best" in this measure. Kraemer is too conservative; it almost always covers the true system reliability far below its value (conservative here is not desirable since actual coverage is far from nominal coverage). The other two bounds yield intermediate results.

In Table 5 we compare the interquartile (IQR) range results (i.e. the range between the 25th (P. 25) and the 75th (P. 75) percentiles of the distribution). Given the skewness/kurtosis of the (small sample) distributions (see the histograms in Fig. 1 through 4) this seems a better measure of the dispersion of the bound than the CV of the bound. Notice how, as reliability increases, the variability of the bound decreases, the other factors held constant. Also, as confidence level increases, so does variability of the bound. Finally, notice how Mann–Grubbs bound is somewhat more variable than El Mawazini's.

Figure 5 presents a graphical comparison corresponding to Table 5 (notice how bound variability, portrayed by IQR, decreases as a function of system reliability). Mawazini–Buehler's bound shows the smallest variability (IQR). However, it was also the most optimistic of the three

Table 4. Bound coverage

Cases		System reliability									
Bd	α	0.822	0.849	0.867	0.886	0.904	0.917	0.927	0.936	0.946	0.951
					Case one						
K	0.50	1.000	1.000	1.000	1.000	1.000	1.00	1.00	1.00	0.999	1.00
	0.25	1.000	1.000	1.000	1.000	1.00	1.00	1.00	1.00	1.00	1.00
	0.10	1.000	1.000	1.000	1.000	1.00	1.00	1.00	1.00	1.00	1.00
E	0.50	0.336	0.375	0.366	0.410	0.406	0.381	0.371	0.415	0.432	0.422
	0.25	0.577	0.614	0.637	0.654	0.668	0.624	0.630	0.651	0.617	0.661
	0.10	0.774	0.788	0.812	0.818	0.822	0.786	0.793	0.801	0.813	0.846
G	0.50	0.799	0.865	0.882	0.912	0.928	0.827	0.878	0.891	0.918	0.927
	0.25	0.914	0.952	0.968	0.971	0.982	0.948	0.969	0.972	0.982	0.982
	0.10	0.981	0.989	0.990	0.993	0.997	0.985	0.989	0.995	0.992	0.999
M	0.50	0.527	0.549	0.551	0.562	0.549	0.551	0.560	0.555	0.581	0.561
	0.25	0.757	0.793	0.785	0.788	0.784	0.789	0.783	0.782	0.748	0.775
	0.10	0.903	0.924	0.913	0.919	0.905	0.907	0.903	0.908	0.911	0.917
					Case two						
K	0.50	1.000	1.000	1.000	1.000	1.00	1.00	1.00	1.00	0.988	1.00
	0.25	1.000	1.000	1.000	1.000	1.00	1.00	1.00	1.00	1.00	1.00
	0.10	1.000	1.000	1.000	1.000	1.00	1.00	1.00	1.00	1.00	1.00
E	0.50	0.311	0.359	0.362	0.362	0.356	0.358	0.343	0.373	0.391	0.380
	0.25	0.549	0.574	0.597	0.603	0.619	0.600	0.599	0.603	0.565	0.610
	0.10	0.759	0.776	0.779	0.785	0.773	0.768	0.766	0.768	0.771	0.812
G	0.50	0.778	0.843	0.868	0.881	0.900	0.808	0.855	0.856	0.901	0.909
	0.25	0.905	0.947	0.957	0.967	0.977	0.938	0.956	0.962	0.969	0.974
	0.10	0.980	0.987	0.987	0.990	0.995	0.981	0.987	0.991	0.991	0.996
M	0.50	0.504	0.516	0.516	0.518	0.505	0.517	0.530	0.523	0.526	0.501
	0.25	0.740	0.768	0.762	0.751	0.750	0.767	0.752	0.748	0.709	0.727
	0.10	0.895	0.909	0.899	0.898	0.879	0.901	0.889	0.890	0.883	0.900

Case one: in all cases, testing stopped at 3rd failure.
Case two: in one subsystem, testing stopped at 1st failure.
Bounds: K—Kraemer; E—El Mawazini-Buehler; G—Grubbs (fiducial); M—Mann-Grubbs (approximately optimum).

Table 5. Bound interquartile range

Cases		System reliability									
Bd	α	0.822	0.849	0.867	0.886	0.904	0.917	0.927	0.936	0.946	0.951
					Case one						
	0.50	0.280	0.288	0.232	0.192	0.155	0.245	0.203	0.165	0.110	0.095
K	0.25	0.271	0.268	0.264	0.207	0.162	0.237	0.220	0.161	0.109	0.102
	0.10	0.252	0.269	0.242	0.231	0.181	0.234	0.207	0.168	0.129	0.116
	0.50	0.065	0.054	0.038	0.036	0.028	0.031	0.025	0.022	0.017	0.015
E	0.25	0.084	0.062	0.057	0.044	0.035	0.038	0.034	0.027	0.020	0.020
	0.10	0.101	0.081	0.059	0.053	0.038	0.042	0.036	0.030	0.024	0.021
	0.50	0.084	0.073	0.052	0.050	0.039	0.043	0.035	0.031	0.024	0.022
G	0.25	0.105	0.080	0.074	0.058	0.047	0.051	0.046	0.037	0.028	0.027
	0.10	0.128	0.104	0.079	0.071	0.053	0.057	0.048	0.041	0.034	0.030
	0.50	0.080	0.066	0.046	0.043	0.033	0.039	0.031	0.027	0.020	0.017
M	0.25	0.102	0.076	0.069	0.051	0.041	0.049	0.043	0.032	0.023	0.022
	0.10	0.130	0.103	0.073	0.065	0.046	0.055	0.048	0.037	0.030	0.026
					Case two						
	0.50	0.248	0.285	0.226	0.185	0.148	0.236	0.194	0.157	0.104	0.089
K	0.25	0.276	0.267	0.260	0.201	0.156	0.230	0.212	0.154	0.104	0.097
	0.10	0.260	0.270	0.239	0.225	0.175	0.228	0.201	0.161	0.123	0.110
	0.50	0.065	0.054	0.037	0.036	0.028	0.031	0.025	0.022	0.017	0.015
E	0.25	0.083	0.062	0.057	0.044	0.034	0.038	0.034	0.027	0.019	0.019
	0.10	0.102	0.082	0.060	0.053	0.039	0.042	0.036	0.030	0.024	0.021
	0.50	0.084	0.073	0.051	0.050	0.039	0.043	0.035	0.031	0.024	0.022
G	0.25	0.105	0.080	0.075	0.059	0.047	0.051	0.046	0.037	0.027	0.027
	0.10	0.128	0.105	0.079	0.071	0.052	0.057	0.049	0.041	0.034	0.030
	0.50	0.081	0.066	0.047	0.043	0.033	0.038	0.031	0.026	0.020	0.017
M	0.25	0.102	0.077	0.070	0.052	0.041	0.049	0.043	0.033	0.023	0.022
	0.10	0.130	0.105	0.073	0.065	0.046	0.055	0.047	0.037	0.030	0.026

Case one: in all cases, testing stopped at 3rd failure.
Case two: in one subsystem, testing stopped at 1st failure.
Bounds: K—Kraemer; E—El Mawazini–Buehler; G—Grubbs (fiducial); M—Mann–Grubbs (approximately optimum).

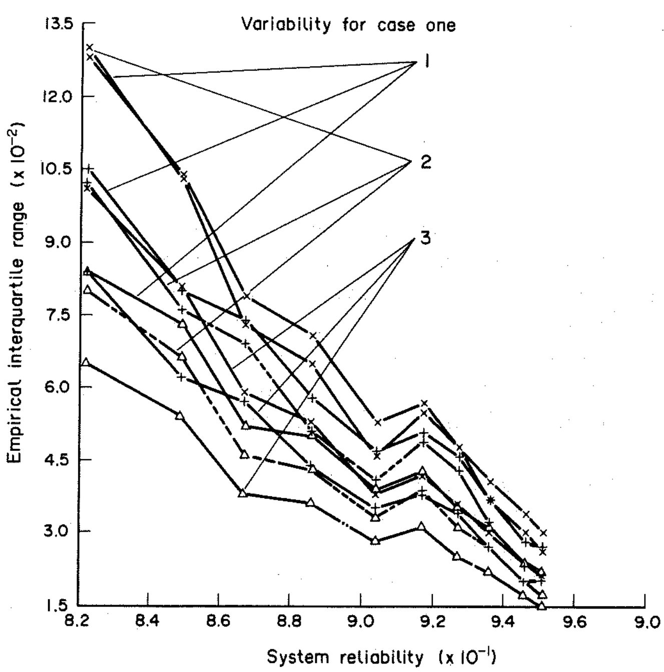

Fig. 5. Bound variability as a function of reliability. 1, Grubbs; 2, Mann–Grubbs; 3, El Mawazini–Buehler.

Table 6. Covariance analysis for bound coverage

Response variable	R^2	Partial F values (coefficient sign) Reliability	Truncation	Structure	Total F
Coverage					
Mann–Grubbs	0.57	1.49 (−)	21.48 (+)	N.S.	—
Bound	0.56	—	20.82 (+)	N.S.	—
Alpha = 0.10	0.04	0.70 (−)	—	—	—
Coverage					
Mawazini–Buehler	0.57	4.63 (+)	15.58 (+)	1.30 (+)	—
Bound	0.45	—	12.84 (+)	1.34 (−)	—
IQR					
Mawazini–Buehler	0.98	175.03 (−)	—	11.83 (−)	193.17
Bound	0.53	—	—	9.30 (+)	—
Grubbs	0.98	153.02 (−)	—	8.64 (−)	176.33
Bound	0.55	—	—	9.98 (+)	—
Mann–Grubbs	0.98	188.79 (+)	—	16.64 (−)	194.48
Bound	0.50	—	—	8.18 (+)	—

best bounds i.e. it covered the real system reliability less often than required by its confidence coefficient, and this is not desirable. Mann–Grubbs is second best in this performance measure.

Histograms in Figs 1–4 also show the empirical modes of the four bounds. Notice how El Mawazini–Buehler's and Mann–Grubb's modes are very close to each other and to the true system reliability. Mann–Grubb's histogram is more skewed to the left, providing better coverage than El Mawazini's. Grubb's histogram is flatter and Kraemer's is almost flat.

Finally, in Table 6 we present the analysis of covariance. Coverage and IQR were regressed on system reliability, truncation method and system structure. We wanted to assess the statistical significance of these factors, both on the coverage and its variability. Notice how, when early truncation is present, it is the only significant factor affecting bound coverage. Hence, number of failures observed is the key factor for coverage, especially when this number is small.

In the lower part of Table 6 we show regression results of IQR on structure, with/without system reliability included as a factor. Reliability structure (see Section 3, part (ii)), was significant, but system reliability was a much more significant factor in the variability of the bounds. We believe, from these regression analyses that, with the appropriate caveats, our experimental results will also hold in other systems supporting the same assumptions of Table 2. In particular, our analysis approach will hold for series configurations. These results justify our experiment.

5. DISCUSSION

In this paper we compare four asymptotic approximations to the exact (El Mawazini) system Reliability bound. Their exact small sample distributions are unknown, hence it is not possible to theoretically investigate or compare them for the small sample case. In previous studies, these bounds had been compared, based on a few cases, with the equivalent exact bound of El Mawazini. We have used, instead, Monte Carlo and a large number of cases. This is the main difference between our work and other bound comparisons previously performed in the literature. Through it, we have attempted to provide the reliability practitioner with more insight on bound coverage and variability by (numerically) characterizing their (small sample) distribution. Using this information, we have compared these four bound approximations, studying its robustness to certain assumption departures and ranking them, accordingly. This is the contribution of our research.

When these bound approximations must be used, the above regression analysis shows that our results are applicable with the pertinent qualifications regarding system configuration and structure. Our findings can provide the practitioner with grounds for qualifying his/her results and understanding their implications. For high cost systems whose structures and configurations are very different from the one we have used in our experiment, a (restricted) similar study could be undertaken. In such a case, our paper provides a roadmap to this approach. The implications involved in assumption violations or relaxations, can be investigated and the practitioner can support his/her confidence in the bound, etc. Such is a midway procedure between the automatic application of any of the above system bound approximations and the extensive (and exclusive) use of simulation as the only reliability assessment tool.

6. CONCLUSIONS

Given the following two caveats:

(1) The "representative" system upon which the present experiment was implemented may have some (minor) effect on the results described below. However, from the regression analysis, and the rationale upon which the "representative" system was conceived, we believe that our results will hold in systems fulfilling the assumptions in Table 2.

(2) The alternative to defining a specific system and implementing an experiment such as ours is working with the exact small sample distribution of the bound approximation. This alternative is not available at present.

(I) General results of system reliability with respect of the four bounds compared:

(i) coverage increases mildly with increasing reliability
(ii) coverage approaches nominal value as confidence coefficient increases
(iii) variability of the bounds are affected by variations in the reliability of any subsystem, and
(iv) variability of the bound decreases both with increasing system reliability and confidence coefficient.

(II) Specific results for Mann–Grubbs "approximately optimal" bound, the preferred among the four analyzed here:

(i) point estimation of the reliability bound:
—closest value to nominal coverage
—closest mode to true system reliability
(ii) variability of the bound point estimation:
—comparatively small variability, among the bounds, and
—relatively stable with respect to changes in system structure.

7. SUMMARY

We recommend Mann–Grubbs bound, when estimating total system reliability from subsystem data for:

(1) it gives the best (coverage) approximation to the exact reliability bound
(2) it captures subsystem reliability changes that affect the reliability of the total system
(3) it is reasonably robust to early truncation and system structure, among the bounds studied, and,
(4) it is easy to understand/implement, well documented and the last in a sequential development/refinement process of system bound approximations.

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