SMALL SAMPLE EMPIRICAL CRITICAL VALUES FOR MULTIVARIATE NORMALITY TESTS

Jorge Luis Romeu, State College at Cortland, NY 13045.
jromeu@suvm.acs.syr.edu

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1.0 Introduction

This paper summarizes an investigation of the empirical critical values (ecv) obtained in a Monte Carlo power study of ten multivariate normality (MVN) goodness of fit (GOF) tests. Eigth of these tests were: Mardia's Skewness, Mardia's Kurtosis, Royston's W, Malkovich and Afifi's, Koziol's Chi Square, Koziol's Angles, Cox and Small's and Hawkin's goodness of fit tests. The other two were versions of a new graphical test (Ozturk and Romeu, 1992).

Some of the GOF tests statistics used critical values (CV) derived from asymptotic distributions (e.g. Mardia's Skewness and Kurtosis tests) which are valid only for large samples. Other tests statistics were purely empirical in nature (e.g. Malkovich and Afifi). Hence, to compare their small sample efficiency, their empirical critical values had to be obtained by simulation.

Large discrepancies between the small sample ecv and the corresponding asymptotic values were soon discovered. Other problems such as dependence on correlation, on hardware used, and algorithm, were also detected (Romeu, 1992a). Such problems were examined in an exhaustive simulation study (Romeu, 1990).

The simulation experiment considered p=2(1)6(2)10 p-variate distributions with two covariance matrices. One matrix had high inter p-variate correlation (0.9) and the other, low inter correlations (0.5). The sample sizes considered were of n=25(25)200.

To evaluate the ten GOF test powers, twelve non normal alternative distributions, ranging from purely skewed to purely kurtic (including combined skewed-kurtic) were used (Romeu and Ozturk. 1992). The validation of this power study is discussed in Romeu, 1992c.

The present paper analyzes the empirical critical values in two of these cases. And the reader is referred to the bibliography for additional information and sources.

2.0 Discussion.

Tables 1 and 2, appended, correspond to the case of p=8 variables, for both mid (0.5) and high (0.9) inter p-variate correlation.

In both tables, the ecv corresponding to seven of the eight MVN GOF tests, are shown by sample size (Cox and Small's test was only compared for the case p=2). Mardia's Kurtosis test is two sided. Hence, both lower/upper ecv are given.

The ecv are also given for sample sizes n=25(25)200, for the 90th, 95th and 99th percentiles (except for Malkovich and Afifi's test, where they correspond to the 1st, 5th and 10th percentiles).

The ecv were obtained through a research grant at the supercomputer in Cornell Theory Center. Five thousand replications were run for each case utilizing the IMSL random number generator.

The ecv's have several potential uses. The first one is to provide more accurate point estimators than the asymptotic distribution when the samples are small. Hence, asymptotic results are invalid. An example of such use is given in Romeu, 1992.

The relevance of this information becomes apparent by constrasting the results in these two tables. The ecv's in Mardia's Skewness and Kurtosis, Malkovich and Afifi's, Koziol's Chi Square and Koziol's Angles tests vary significantly with the sample sizes. Hence, using the asymptotic critical values when samples are small can lead to serious Type I errors. A statistical assessment of the rate of convergence to their asymptotic values is given in Romeu 1992a.

The second use is to provide point estimators for the ecv's in the empirical tests, avoiding the need of the practitioner to calculate these point estimators by Monte Carlo techniques.

The number of p-variates (or degrees of freedom) affected Mardia's Skewness, Royston's W and Koziol's Angles test statistics ecv. The remaining test statistics ecv were not significantly affected by the number of p-variates. The effect of numerical algorithms (and related hardware precision problems) were significant in Koziol's Angles and Royston's W tests. Koziol's test requires the numerical calculation of eigenvalues and eigenvectors, as well as inverting matrices, both in the numerator and denominator. Significant differences were detected between the results obtained in an IBM 3090 and in the more accurate supercomputer at Cornell. Royston's test included a correction for correlation that never achieved to completely remove the inter p-variate correlation.

One important aspect of inter p-variate correlation is that, in practice, the true covariance matrix is seldom known. This research shows how Royston's W and Koziol's Angles tests are significantly affected by correlation. Tables 1 and 2, rho=0.5,0.9, also show how Malkovich and Afifi's, Mardia's Skewness and Kurtosis and Koziol's Chi Square tests are robust to correlation effect. Cox and Small's and Hawkins' tests are midldly affected. Examples of these statistical analyses are presented in Romeu 1992a.

Finally, the ecv can also help establish adequate sample sizes for using the asymptotic critical values. Non parametric 95% confidence intervals for the ecv were obtained in Romeu 1992a and 1992c.

The asymptotic critical values were assessed using 95% confidence intervals. Mardia's Skewness test ecv cover the asymptotic critical values for sample sizes n=100. Similar confidence intervals from Mardia's Kurtosis ecv's do not cover the asymptotic critical values until samples are larger than 200. Mardia's Kurtosis is known to converge much slower than Mardia's Skewness test.

3.0 Summary.

The present research has provided several theoretical and practical results:

(i) small sample point estimators (ecv) of the critical values of asymptotic MVN GOF tests were obtained. These empirical estimators can be used when the samples are small and the asymptotic results cannot be applied. A complete set of ecv tables can be found in Romeu 1990 or Romeu and Ozturk, 1992.

(ii) similarly, ecv for empirical MVN GOF tests were obtained, for several combinations of sample sizes and p-variates. This saves the practitioner of having to obtain them by Monte Carlo, at every instance.

(iii) minimal sample sizes beyond which asymptotic critical values are valid. For Mardia's Skewness and Kurtosis tests, these sizes are, respectively, n=100 and n>200.

(vi) inter p-variate correlation effect in MVN GOF tests were assessed. A significant problem arises when the covariance matrix is not known. Inter p-variate correlation is shown not to be a major problem in either of the two Mardia's tests.

4.0 Acknowledgement.

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TABLE NO. 1 CRITICAL VALUES FOR THE CASE P = 8 VARIATES.

RHO=0.5 N	%	SKEWNESS TEST	KURTOSIS LOWER	KURTOSIS UPPER	ROYSTON W	MALKOVICH AFIFI	KOZIOL CHI-SQR.	COX-SMAL REG	HAWKINS TEST	KOZIOL ANGLES
25	90	107.76	-1.87	-0.28	10.47	0.731	0.193		0.914	11.09
25	95	113.34	-1.97	-0.07	12.80	0.705	0.233		1.130	13.50
25	99	124.91	-2.17	0.37	18.66	0.647	0.327		1.700	18.92
50	90	125.36	-1.93	0.40	10.92	0.876	0.162		0.936	13.02
50	95	131.86	-2.11	0.69	13.21	0.862	0.203		1.144	15.49
50	99	145.86	-2.37	1.26	18.97	0.829	0.301		1.676	21.06
75	90	131.03	-1.92	0.75	11.04	0.922	0.155		0.950	13.67
75	95	138.05	-2.09	1.09	13.24	0.912	0.194		1.156	16.27
75	99	153.91	-2.42	1.69	18.17	0.890	0.283		1.654	22.50
100	90	132.73	-1.91	0.816	11.19	0.945	0.152		0.933	13.77
100	95	139.23	-2.13	1.16	13.42	0.938	0.191		1.148	16.39
100	99	153.34	-2.48	1.84	19.07	0.923	0.279		1.659	22.92
125	90	135.25	-1.90	0.98	11.39	0.956	0.150		0.938	14.06
125	95	142.39	-2.10	1.35	13.63	0.951	0.188		1.172	16.65
125	99	156.19	-2.54	2.09	17.91	0.938	0.274		1.656	22.26
150	90	135.87	-1.90	1.02	11.26	0.964	0.152		0.954	13.78
150	95	142.33	-2.15	1.41	13.68	0.960	0.189		1.155	16.27
150	99	156.59	-2.58	2.41	18.75	0.950	0.273		1.650	22.56
175	90	136.26	-1.88	1.15	11.43	0.969	0.148		0.945	13.96
175	95	142.69	-2.09	1.43	13.70	0.966	0.185		1.140	16.49
175	99	156.75	-2.61	2.07	18.76	0.957	0.265		1.637	21.45
200	90	137.64	-1.87	1.13	11.59	0.973	0.148		0.941	14.15
200	95	144.35	-2.13	1.48	14.02	0.970	0.183		1.141	16.56
200	99	156.43	-2.57	2.13	19.09	0.963	0.263		1.600	22.83

TABLE NO. 2 CRITICAL VALUES FOR THE CASE P = 8 VARIATES.

RHO=0.9 N	%	SKEWNESS TEST	KURTOSIS LOWER	KURTOSIS UPPER	ROYSTON W	MALKOVICH AFIFI	KOZIOL CHI-SQR.	COX-SMAL REG	HAWKINS TEST	KOZIOL ANGLES
25	90	108.24	-1.87	-0.30	7.24	0.730	0.190		0.915	12.52
25	95	113.39	-1.98	-0.09	9.09	0.705	0.237		1.121	15.22
25	99	123.51	-2.15	0.28	13.82	0.646	0.320		1.637	22.72
50	90	124.68	-1.90	0.38	7.45	0.876	0.157		0.921	14.63
50	95	131.71	-2.07	0.70	9.22	0.862	0.194		1.124	17.87
50	99	146.41	-2.39	1.29	13.62	0.828	0.282		1.595	26.21
75	90	130.50	-1.89	0.72	7.59	0.923	0.154		0.936	15.01
75	95	137.95	-2.10	1.05	9.49	0.913	0.190		1.145	18.63
75	99	152.86	-2.43	1.73	13.55	0.892	0.271		1.680	26.53
100	90	133.09	-1.91	0.86	7.67	0.944	0.153		0.937	15.33
100	95	140.09	-2.11	1.17	9.35	0.937	0.189		1.145	18.77
100	99	154.11	-2.51	1.83	13.37	0.922	0.270		1.617	26.67
125	90	134.95	-1.89	0.95	7.72	0.956	0.147		0.918	15.64
125	95	141.59	-2.09	1.29	9.57	0.951	0.183		1.134	19.09
125	99	154.96	-2.52	1.94	14.31	0.939	0.259		1.604	27.56
150	90	135.60	-1.89	1.03	7.66	0.964	0.148		0.931	15.78
150	95	142.35	-2.11	1.42	9.61	0.959	0.183		1.126	19.39
150	99	155.33	-2.55	2.20	13.76	0.950	0.265		1.645	27.64
175	90	137.05	-1.85	1.12	7.66	0.969	0.147		0.933	15.35
175	95	144.38	-2.08	1.46	9.68	0.965	0.181		1.158	18.70
175	99	158.84	-2.51	2.06	14.34	0.957	0.276		1.660	27.50
200	90	136.58	-1.90	1.14	7.73	0.973	0.149		0.931	15.69
200	95	143.35	-2.13	1.50	9.41	0.970	0.184		1.164	18.92
200	99	156.15	-2.62	2.12	14.13	0.963	0.273		1.635	25.57

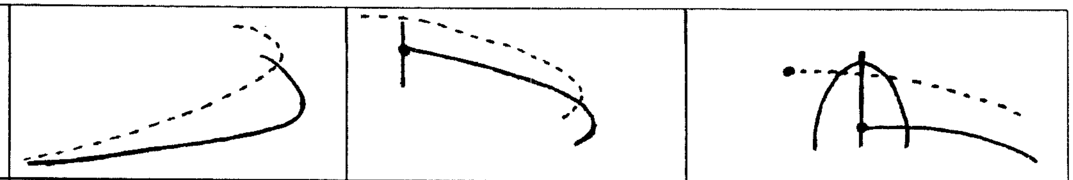
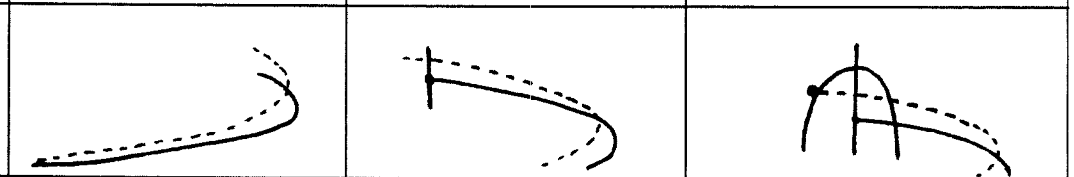
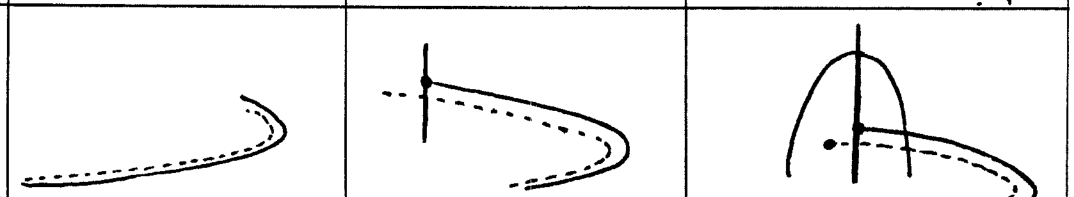
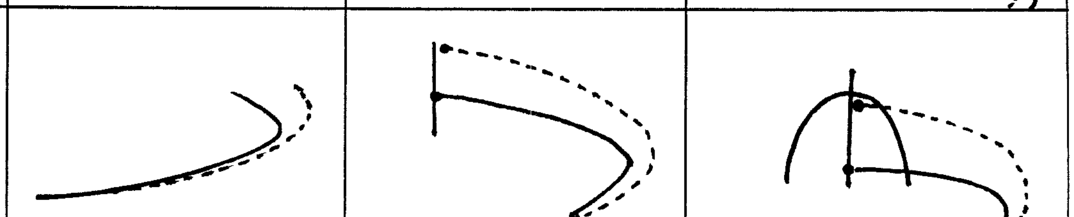
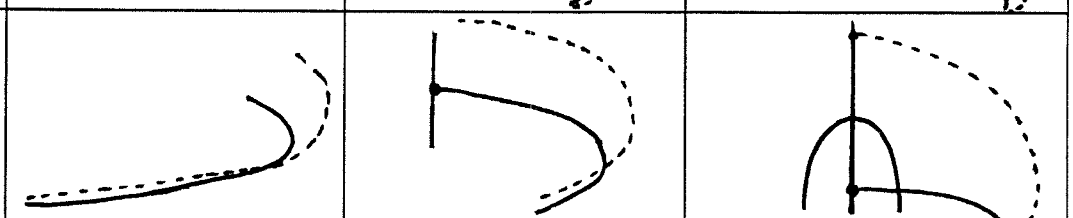
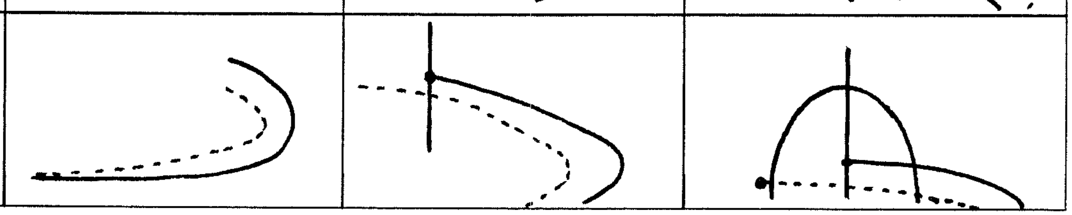
Shape	Distrib.	Vector Patterns: 1st. Half	2nd. half	Endpoint
Skewed	GLD - 1			
	GLD - 3			
Kurtic	t (8)			
	GLD - 2			
	UNIFORM			
Both	Chi Square			

Table I: Empirical Powers of Nine Multivariate Normality Tests.

PERCENT REJECTIONS FOR N= 2500 TOTAL CASES.

METHOD:	ALPHA=0.10	ALPHA=0.05	ALPHA=0.01
CHOLESKI	0.37640	0.25240	0.07440
SIGMA	0.13680	0.08160	0.01800
M-SKEW	0.00040	0.00040	0.00000
M-KURT	0.79560	0.72040	0.43960
ROYSTONW	0.48720	0.28680	0.11560
MALKOV	0.00760	0.00120	0.00000
KOZ-CHI	0.81800	0.72160	0.44600
KOZANGLE	0.00000	0.00000	0.00000
HAWKINS	0.68800	0.57080	0.28920

PERCENT REJECTIONS FOR N= 2500 TOTAL CASES.

METHOD:	ALPHA=0.10	ALPHA=0.05	ALPHA=0.01
CHOLESKY	0.76360	0.60920	0.31000
SIGMA	0.61960	0.44240	0.18360
M-SKEW	0.65320	0.50600	0.18560
M-KURT	0.17840	0.11400	0.02920
ROYSTONW	0.92880	0.85360	0.74120
MALKOV	0.15880	0.08920	0.01680
KOZ-CHI	0.06800	0.03160	0.00680
KOZANGLE	0.07320	0.02320	0.00960
HAWKINS	0.20480	0.13360	0.04040

Legend:

CHOLESKI: Our Q_n (Cholesky Implementation)
SIGMA: Our Q_n (Sigma Inverse Implementation)
MSKEW: Mardia's Skewness Test
MKURT: Mardia's Kurtosis Test
MALKOV: Malkovich and Afifi's Test
ROYSTON: Royston's Test
KOZCHI: Koziol's Chi Square Test
KOZANGL: Koziol's Angles Test
HAWKINS: Hawkins' Test