

Covid-19 ICU Staff and Equipment Requirements using the Negative Binomial

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1.0 Introduction

We use statistical distributions to estimate the *Covid-19 Staff and Equipment requirements* to successfully cope with an overflow of patients. This work is part of *our struggle vs. Covid-19*: <https://www.researchgate.net/publication/341282217> A Proposal for Fighting Covid-19 and its Economic Fallout Previous *work* includes screening with Design of Experiments: <https://www.researchgate.net/publication/344924536> Design of Experiments DOE in Covid-19 Factor Screening and Assessment using statistical methods to establish a new *Vaccine Life*: <https://www.researchgate.net/publication/344495955> Survival Analysis Methods Applied to Establishing Covid-19 Vaccine Life as well as to help *accelerate vaccine testing*: <https://www.researchgate.net/publication/344193195> Some Statistical Methods to Accelerate Covid-19 Vaccine Testing and a *Markov model to study problems of reopening college*: <https://www.researchgate.net/publication/343825461> A Markov Model to Study College Re-opening Under Covid-19 and the *effects of Herd Immunization*: <https://www.researchgate.net/publication/343345908> A Markov Model to Study Covid-19 Herd Immunization?channel=doi&linkId=5f244905458515b729f78487&showFulltext=true as well as of *general survival*: <https://www.researchgate.net/publication/343021113> A Markov Chain Model for Covid-19 Survival Analysis about socio-economic and racial issues affected by Covid-19: <https://www.researchgate.net/publication/343700072> A Digression About Race Ethnicity Class and Covid-19 and developing *A Markov Chain Model for Covid-19 Survival Analysis*: <https://www.researchgate.net/publication/343021113> A Markov Chain Model for Covid-19 Survival Analysis and *An Example of Survival Analysis Applied to analyzing Covid-19 Data*: <https://www.researchgate.net/publication/342583500> An Example of Survival Analysis Data Applied to Covid-19, and *Multivariate Statistics in the Analysis of Covid-19 Data*, and *More on Applying Multivariate Statistics to Covid-19 Data*, both of which can also be found in: <https://www.researchgate.net/publication/341385856> Multivariate Stats PC Discrimination in the Analysis of Covid-19, and the implementation of *multivariate analyses* methods such as: <https://www.researchgate.net/publication/342154667> More on Applying Principal Components Discrimination Analysis to Covid-19 Design of Experiments to the Assessment of Covid-19: <https://www.researchgate.net/publication/341532612> Example of a DOE Application to Coronavirus Data Analysis *Offshoring*: <https://www.researchgate.net/publication/341685776> Off-Shoring Taxpayers and the Coronavirus Pandemic and *reliability methods* in ICU assessment: <https://www.researchgate.net/publication/342449617> Example of the Design and Operation of an ICU using Reliability Principles and *Quality Control methods for monitoring Covid-19*: <https://web.cortland.edu/matresearch/ApplicatSPCtoCovid19MFE2020.pdf>

2.0 Problem Statement

We use Poisson and Negative Binomial distributions to *estimate Covid-19 Staff and Equipment requirements* to successfully *cope with a possible patient overflow*. We assume that the reader is familiar with our previous paper on *designing and operating a hospital ICU*, that is found in: https://www.researchgate.net/publication/342449617_Example_of_the_Design_and_Operation_of_an_ICU_using_Reliability_Principles There, we discussed how to assess ICU and ventilator reliability and maintainability requirements using survival analysis, FTA & FMEA methodology. Also, mean life, times to failure, and confidence intervals of life parameters were obtained.

In the present article *we implement several statistical procedures to estimate health care system (hospital, ward, ICU, etc) load, and operating requirements (number of beds, of doctors, nurses, ventilators, etc.) to successfully cope with a possible health system overload, during the second Covid-19 wave*. One of the important concerns of the second wave is that the health care system may have to handle a very large number of patients requiring critical medical attention.

We assume that data from the first Covid-19 wave is available, and that there does exist a data collection system in place, to update said data base with incoming data from the second wave. Lack of data is one of the most daunting problems in correctly estimating system requirements.

After establishing the number of incoming Covid-19 cases using said data base, we estimate the operating requirements for successfully dealing with increasing patient input. We *first* use the *Poisson distribution*, a more traditional procedure for these endeavors. *Secondly*, we implement *the Negative Binomial* distribution, which is not so commonly used here, but which yields good results. We then *validate* the results *by simulating patient admissions* with the Negative Binomial and the estimated parameters, *evaluating the results* through a *survival analysis*. We then *compare and discuss* the different methods' evaluations and we conclude.

3.0 Poisson Distribution

Assume that a hospital ICU has calculated how, every hour, on the average, five new Covid-19 patients are admitted. A Poisson distribution¹ with Mean five (Lambda: $\lambda = 5$) has been used to describe the current situation. It is customary to provide *three estimations: an average, a best, and a worst case*. We use the mean point estimator ($\lambda=5$) for the average case, and the Lower and Upper Limits of the Confidence Interval (CI) for the mean, as best and worst case values.

There are *two situations*. First, both the average and the data that provided it exist. Secondly, said *average* comes from a *subject-matter expert* estimated guess. We will illustrate both cases next.

We *generate Covid-19 patient hourly admissions data for 50 consecutive hours* (over two days) at a hospital ward, *using a Poisson* distribution with *mean $\lambda = 5$* :

8 5 2 4 4 7 4 7 3 7 4 5 5 5 4 7 7 6 3 5 3 4 5 4 5
10 4 7 7 8 2 5 4 2 4 9 6 6 10 7 4 3 6 4 4 5 9 2 7 6

¹ Poisson Distribution: <https://www.itl.nist.gov/div898/handbook/eda/section3/eda366j.htm>

These *simulated data* represent a data collection effort of 50 hours at a hospital ward. There, the admissions of Covid-19 patients per hour were obtained. Data were then processed, to obtain the *point estimator* λ and *confidence interval* for the unknown Poisson Mean λ .

There are several procedures to obtain a CI for a Poisson Mean. Some are given in:
<http://onbiostatistics.blogspot.com/2014/03/computing-confidence-interval-for.html>

We used the *large sample approach*. We first computed the Poisson *sample mean* $\lambda=5.28$. Then, we obtained the sample mean *Standard Deviation*: $\text{Sqrt}(5.20/50) = 0.3225$. Then we add/subtract said Std-Dev, times the 97.5th percentile of the Normal Standard (1.95), to mean λ , obtaining:

95% CI: Lower Limit: 4.64792; Upper Limit: 5.91208

We calculated the *Average, Best and Worst Cases Probabilities* for selected *admissions per day*:

Best Case: Prob. up to 130 admissions in 24 hours: Poisson ($4.64*24=111.55$; 130) = 0.96

Prob. (admitting over 130 patients) = $1 - 0.96 = 0.04$

Average Case: for up to 130 admissions in 24 hours: Poisson ($5.28*24=126.72$; 130) = 0.64

Prob. (admitting over 130 patients) = $1 - 0.636 = 0.364$

Worse Case: for up to 130 admissions in 24 hours: Poisson ($2.91*24=141.8$; 130) = 0.17

Prob. (admitting over 130 patients) = $1 - 0.17 = 0.83$

We can then *use such probabilities* of Covid-19 admissions *to help staff doctors and nurses*, to prepare or seek beds, ventilators and other necessary hospital equipment, etc. *For example*, if we select an *Average Case of 130 admissions*, then *64% of the time actual admissions will go over 130*. It would be safer to plan for a larger number of admissions that would be surpassed say 5%:

Average Case: for up to 145 admissions in 24 hours: Poisson ($5.28*24=126.72$; 145) = 0.9499

Prob.(admitting over 145 patients) = $1 - 0.9499 = 0.05$ (or 5% of the time)

Planning for admission of 145 Covid-19 daily patients, surpassed 5% of the days, is *much safer*.

When there is no data on hospital hourly admissions, or when there is an estimate, but the data used to obtain it is lost, we *use a subject matter expert* to provide an estimate λ of the Poisson Mean, as well as values for the Best and Worst cases. These do not yield the best results, but can be *used as initial estimates*, and can be *later updated* as new information is collected.

4.0 Negative Binomial Distribution

In the above-mentioned case if *not having estimations or a CI for the Poisson Mean* we may use *another procedure*, which may provide better information. We will require knowledge about (or an estimation of) the *probability "p" of hospital bed occupancy*, per a pre-established interval of time (e.g. per day, per hour, half-hour). Let's explain through *a numerical example*.

Assume that a hospital has investigated how, in every *pre-established interval of time* (e.g. an hour, a half-hour, 15 minutes), a Covid-19 *patient has (or has not) be admitted*. Assume that

each interval constitutes a *Bernoulli trial* with two outcomes: (1) a patient is admitted during this time (2) or is not. Such two results are denoted with a 1 or a 0. We count the number of intervals (or Bernoulli trials) until a patient is admitted, considered a success with probability p, or is not (a failure; with 1-p). The distribution used to describe this situation is the *Geometric*² (p).

We then consider the number of time intervals (X) in an eight-hour shift (i.e. the number, out of its eight time intervals) until a total of K successes or admissions, is obtained. The statistical distribution used to describe this situation is the *Negative Binomial* (x,k,p)³, which has three parameters: success probability “p”, number of intervals “x”, and desired successes, “k”.

For example, consider the run of “x” hour-long time intervals, where no patient admissions has occurred (failures), until one admission (success) finally occurs. Then, consider the number (x) of time intervals until the fifth patient (k=5) is admitted. We can have two different (one individual and one cumulative) Negative Binomial probability statements:

Probability that the Kth admission occurs at the Xth time slot: $P\{X=x; k\}$

Probability that Kth (but no more) admission occurs, at or before the Xth time slot: $P\{X \leq x; k\}$

This modeling approach may not be as traditional in Public Health. But we believe it provides useful information. We will next illustrate its implementation, through a numerical example.

Fifty observations with failures (0) and successes (1), each one representing either a Covid-19 admission (or not) at a hospital ward, were generated for fifty time intervals (Bernoulli trials) of one hour each, using probability of success $p = 0.2$ per hourly time interval. Said data are:

1 0 1 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 0
0 1 0 1 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0

Said simulated data represent a data collection effort of 50 hours at a hospital ward. Zero or unit represent the admissions (or not) of Covid-19 patients, in each hourly interval. From these data we obtained a point estimator and confidence interval for the unknown Bernoulli proportion p.

In order to provide the three customary estimations (average, best and worst cases) we use the point estimator $p=0.2$ as the Average case, and the Lower and Upper Limits of its CI as the best and worst case values. The procedure we followed to calculate a CI for ‘p’ is described in: <https://stats.stackexchange.com/questions/4756/confidence-interval-for-bernoulli-sampling>

We used the large sample approach. We first obtained the sample proportion $p = 0.2$. Then, we obtained the Standard Deviation for proportion p: $\text{Sqrt}(0.2*0.8/50) = 0.0565$. Then we added and subtracted to proportion $p=0.2$, said Std-Dev times the Normal Standard 97.5th percentile (1.95):

95% CI: Lower Limit: 0.0891; Upper Limit: 0.3109

² Geometric Dist. parameters, formulas, etc.: https://en.wikipedia.org/wiki/Geometric_distribution

³ Negative Binomial Dist. parameters, formulas, etc.: https://en.wikipedia.org/wiki/Negative_binomial_distribution

We give details for the *Average case* below. All cases are implemented in a similar fashion.

Since every hour a Covid-19 patient is either admitted (1) or not (0), we *consider the run* of all hourly time slots (X) *until the Kth desired admission* (success) occurs. The time slot events are independent. *Probability* that the Third patient is admitted in the Fifth hourly time slot is:

$$\text{Probability (Third Patient admitted in 5th slot)} = P\{X=5;k=3;p\} = (1-p)^{5-3} * p^3 = 0.8^2 * 0.2^3 = 0.0307$$

$$\text{Probability (Third Patient admitted up to 5th slot)} = P\{X \leq 5;k=3;p\} = \sum_i P\{X=i\} = 0.0579$$

Prob. (admitting the Third patient after the Fifth time slot) = 1- 0. 0579 = 0.9421

Scheduling of beds, doctors, nurses, ventilators etc. is dependent on the number of admissions. That 94.2% of the times, there will be more admissions than three is a very risky situation. One way to decrease such risk is to increase the time X to admission from Five to Eight times slots:

The *probability* that the Third Covid-19 patient is admitted up to the last (8th) hourly shift slot:

$$\text{Probability (X} \leq 8;k=3;p) = \text{Negative-Binomial (x} \leq 8; k=3; p=0.2) = 0.2031$$

Prob. (admitting over 3 patients in an eight hour shift) = 1- 0.2031 = 0.7969

Thence, if we have *at most three* ICU beds, ventilators, etc., *available* in an eight-hour shift, *ICU will be overwhelmed 79.6% of the shifts*, as there will occur over three admissions per shift. ICU will need to *increase available* beds, ventilators, etc., *to cope with such p=0.2 admissions rate.*

There is another *application of the Negative Binomial, to help with Logistics of Covid-19 patient admissions.* It consists in *finding an admissions pattern* (e.g. a convenient p) *that fits the results* we are observing in the ICU, *and estimating from it the staff etc. requirements* needed to cope.

Assume that we are *observing admissions* of about 20 patients *per eight hour shift.* We want to *explore* this situation further *using the Negative Binomial.* We create a *smaller time slot*, dividing the hourly interval into 60 minutes. We *define p = 0.07, 0.06, 0.05, 0.04* per minute, respectively. For example, *for p=0.06* we have: $0.06 * 15 = 0.9$ yielding a 90% chance of having: one patient admitted every 15 minutes, and of $0.15 * 4 * 8 = 28.8$ *patient admitted per shift*, and so forth.

We calculated the *probabilities of admission for selected minutes* (180 minutes=three hours; 240 minutes=four hours, etc. until 480 minutes = eight hours). Calculation results (Evt are minutes, and NBin04 etc. are the probabilities of having 20 admissions up to Evt minutes) are below:

Evt	NBim04	NBim05	NBim06	NBin07
240	0.00175	0.01848	0.08739	0.24114
300	0.01903	0.11904	0.34626	0.62178
360	0.08964	0.34676	0.67004	0.88332
420	0.24399	0.61999	0.88204	0.97610
480	0.45797	0.82638	0.96871	0.99648

For example, for $p=0.06$, 1/3 of the times (300 minutes=5 hours) there are 20 admissions, or one half admissions by 330 minutes, or 2/3 by 360 minutes (6 hours); or 97% by the shift end (8 hrs).

$$\text{Prob. (admitting over 20 patients, in an 8 hour shift; } p=0.06) = 1 - 0.9687 = 0.0313$$

Thence, if there are 20 ICU beds, ventilators, etc. available per eight-hour shift, and the true rate of admission is $p=0.06$ per minute (case of 28.8 admissions/shift), the ICU will be overwhelmed 3.13% of the shifts. This approach allows public health and hospital professionals to assess, in advance, if there are enough resources to successfully deal with a rising admissions situation. If not, this approach allows them to prepare a plan B by comparing other alternatives created using different numbers of admissions, or of probabilities, per an eight hour shift.

To better illustrate this approach, patient admissions, for $p = 0.04, 0.05, 0.06$ were simulated for 100 shifts. The descriptor statistics are presented below:

Var20	N	Mean	SEMn	StDev	Min	Q1	Med	Q3	Max
NB04	100	505.6	10.1	101.2	289.0	432.3	495.0	564.8	823.0
NB05	100	401.3	8.93	89.3	175.0	346.2	403.0	449.5	656.0
NB06	100	334.9	6.30	63.1	197.0	284.0	332.0	368.5	551.0

Notice how, for smaller admission probabilities (and less patients admitted), the mean time to arrive to 20 admissions is longer (e.g. 505 minutes for $p=0.04$) than for larger probabilities (334 minutes for $p=0.06$). That is why these ICUs fill up sooner. Mean and Median are very close, so these distribution are relatively symmetric. Standard deviations and quartiles may help build some intervals for playing the “what if” game with admission input and assessing their results.

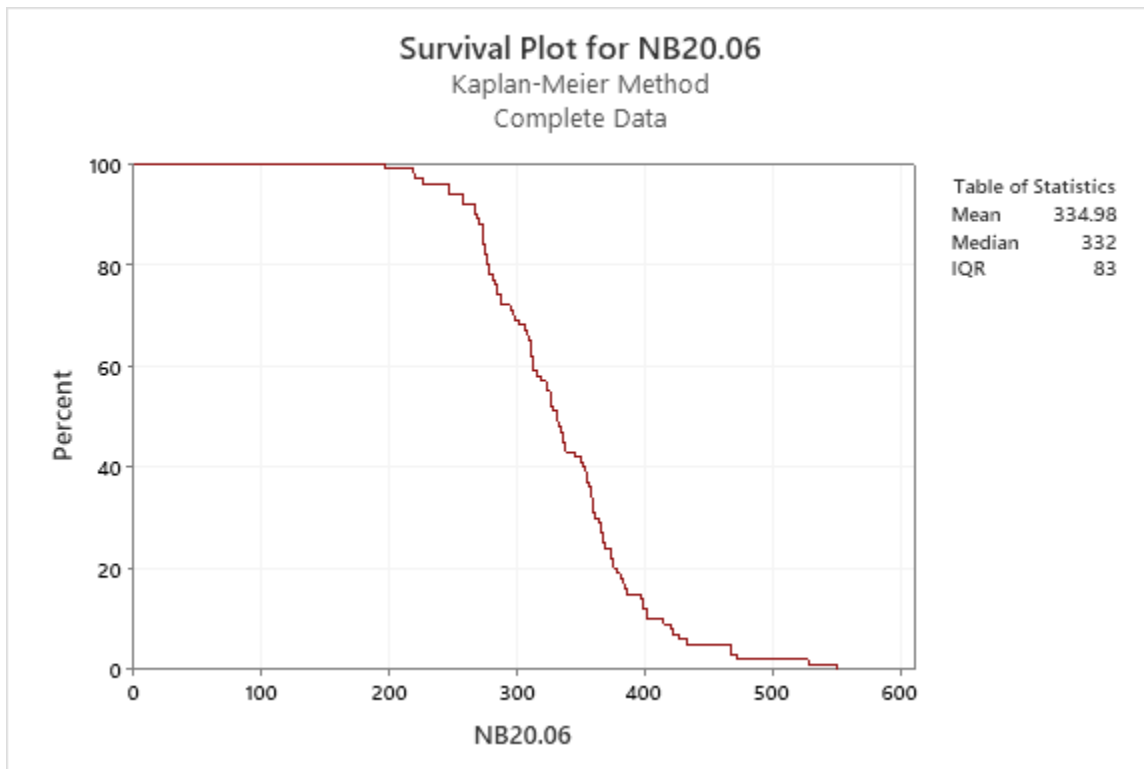
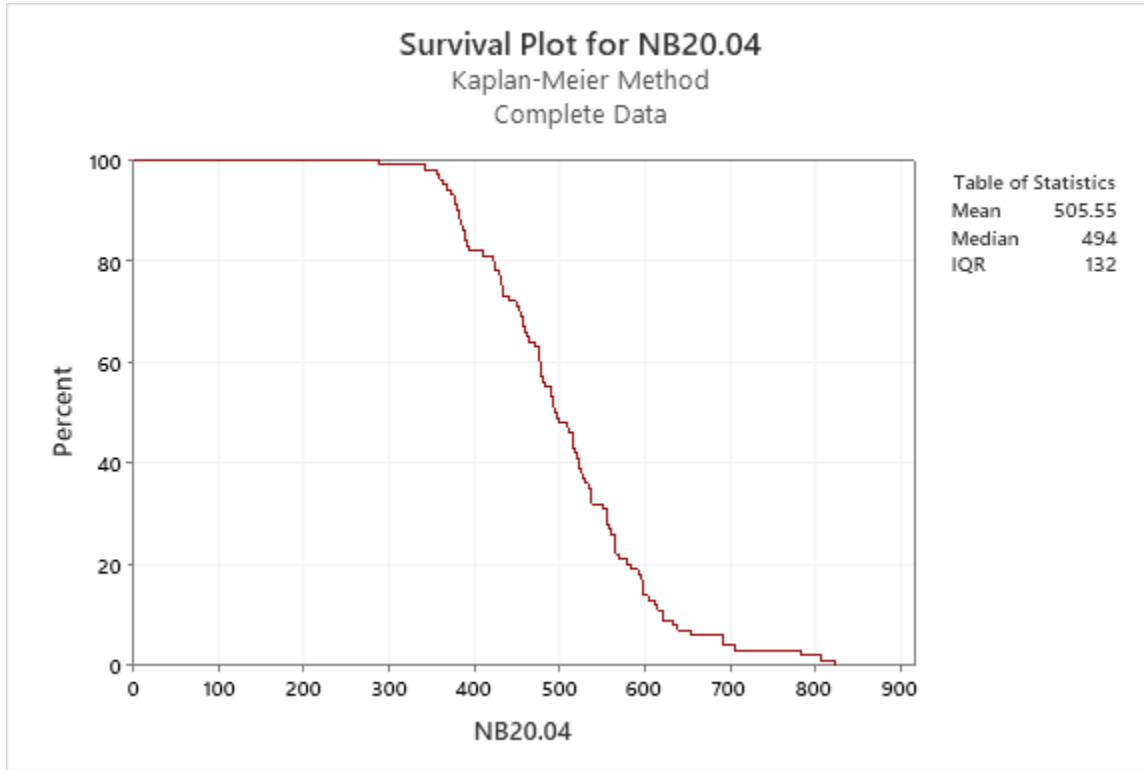
To assess the simulated results as well as provide the above-mentioned standard deviations and quartiles (to play “what if” comparisons) we implemented (Kaplan-Meier) survival analyses. We present selected values of minutes, survival probabilities, and 95% confidence intervals, below:

Mean (min)	Std-Dev	LowerBd	UpperBd	Q1	Med	Q3	IQR	Prob.
505.55	10.1249	485.706	525.394	432	494	564	132	0.04
401.36	8.93191	383.854	418.866	346	401	448	102	0.05
334.98	6.30105	322.630	347.330	284	332	367	83	0.06

Min.	SurvPr.	1-Surv	LowerLim.	UpperLim.	Prob.
359	0.96	0.04	0.921593	0.99841	0.04
481	0.56	0.44	0.462710	0.65729	0.04
520	0.41	0.59	0.313602	0.50640	0.04
359	0.67	0.33	0.577840	0.76216	0.05
480	0.17	0.83	0.096377	0.24362	0.05
515	0.09	0.91	0.033909	0.14609	0.05
360	0.31	0.69	0.219353	0.40065	0.06
472	0.02	0.98	0.000000	0.04744	0.06
528	0.01	0.99	0.000000	0.02950	0.06

Notice how, about 360 minutes (after 6 shift hours), 96%, 67% and 31% of times, these wards or ICUs still haven't had 20 admissions. At shift end (480 minutes) survival rates are 56%, 17% and 2%, respectively and 1-Surv probs are close to the NegBin. ($k \leq 20$). Forty minutes after shifts end (about 520 minutes), rates are 41%, 9% and 1%. If true admission rates were about 20 pat./shift, the 20 bed ICUs will do fine. If admission rates go up to 29, ICUs will be overloaded pretty fast.

We present *below, the survival plots* for $p=0.04$ and 0.06 that extend what has been said above. Select any time (abscissas) and verify the corresponding survival probability (ordinate) value:



5.0 Discussion and Extensions

Methods discussed in this paper can be used to estimate staff and equipment requirements: ICUs, ventilators, doctors, nurses, support personnel, and medical equipment, that increasing number of admissions, stemming from the second wave of Covid-19, will require. Estimations are made by conveniently redefining success and failure events, and their corresponding probabilities, as done above. Such estimations are badly needed at all levels, especially at regional and ICU levels.

In addition, *the length of use of ICUs and its medical resources* is a very important factor in their availability. Readers are directed to two papers on survival analysis, from our previous work: <https://www.researchgate.net/publication/342583500> An Example of Survival Analysis Data Applied to Covid-19 There, probabilities of survival of patients on ventilators, given their age and co-morbidities, are estimated, including providing estimates of their times to death: <https://www.researchgate.net/publication/343021113> A Markov Chain Model for Covid-19 Survival Analysis If resource shortages bring about a Triage, such estimates may be used to determine patient allocation of ward, ICU and ventilator facilities.

We assumed that the admissions processes follow first a Poisson, and then a Negative Binomial distribution. If such assumptions are incorrect, the ensuing results are unsubstantiated. There are ways to assess such assumptions, implementing Goodness-of-Fit tests to the data collected. This is outside the scope of our work, but we include some titles, in the Bibliography, for reference.

The Poisson and Negative Binomial distributions are based on different fundamentals. Poisson is based on the number of events per unit time. If a point process follows the Poisson distribution, then the times between successive inputs are distributed Exponential. The Geometric distribution is concerned with the number of failures until a success occurs. Negative Binomial distribution is obtained from the sum of independent and identically distributed Geometric random variables. It yields the number of events required until a pre-established number of successes occur.

The Geometric distribution is the discrete analogue of the Exponential. The Negative Binomial is the discrete analogue of the Gamma, and is obtained from the sum of independent, identically distributed Exponential random variables. Notice, in the table below, how the Geometric and Exponential distributions, as well as the Negative Binomial and the Gamma distributions, are relatively close, for moderately large values of X;

X	Geo(0.2)	Exp(5)	NB(3,8,.2)	Gamma(3,5)
7	0.790285	0.753403	0.148032	0.166502
8	0.832228	0.798103	0.203082	0.216642
9	0.865782	0.834701	0.261802	0.269379
10	0.892626	0.864665	0.322200	0.323324

The above is not just a pedantic show off about theoretical knowledge, but also a useful fact. Not every statistics package includes these four distributions. For example, older versions of Minitab do not include the Negative Binomial (the newest version does). But they do include the Gamma and Geometric. An alternative is to use the distribution definitions above to directly obtain (or approximate) the Negative Binomial through either Geometric or Gamma, thus avoiding the use of more convoluted procedures, based on the Binomial distribution.

Finally, in the Bibliography section, we included the urls of two excellent statistics textbooks discussing the distributions used in this paper, and an article on combining statistics with O.R.

6. Conclusions

This Covid-19 work stems from *our proposal to the retired* academic and research communities: https://www.researchgate.net/publication/341282217_A_Proposal_for_Fighting_Covid-19_and_its_Economic_Fallout which pursues *one goal: to contribute to defeat Covid-19*.

This paper is a *tutorial on the uses of the Negative Binomial Distribution*, to help estimate the staff and equipment hospitals require, to deal with a surge in the admission of *Covid-19 patients*. The data analyzed was created using this researcher's experience and information. Our numerical results have only illustrative value. However, researchers, public health, and medical officers and practitioners, can *follow these statistical procedures, substituting their data for ours, generating additional analyses, and including new factors, as they become available*.

We want to *reach four audiences: (1) public health* professionals and researchers, (2) *medical doctors*, (3) *statisticians* and (4) *the public* in general. We want to *encourage public health and medical professionals* to use more statistical procedures, not always easy to implement. Health and medical professionals, and statisticians, need to do more joint work: not only after data have been collected, but also at the time that experiments are being designed. Joint work enables the possibility of *extrapolating to the general population (statistical inference)* the promising *results obtained in their laboratories and hospital wards*. This is the final objective of research.

We want to encourage statisticians, especially those retired, who have the experience, financial support (their pension), and the time to provide such assistance, to contribute in helping with the planning, implementation and analysis of statistical procedures –or with writing about them.

We want to *provide illustrative examples* to doctors, public health researchers, and to the general public, to help them better understand what the others do, *fostering more efficient* collaboration.

Finally, we have written a *series of papers on statistical analysis* of Covid-19. They are listed in the initial section of this article, with their *web addresses*. Such papers could become a part of a *biostatistics course* in public health, or an applications course, in the medical *curriculum*.

Bibliography

Beyer, W., Editor. Handbook of Tables for Probability and Statistics. The Chemical Rubber Co. (CRC). Ohio. 1966.

Box, G., Hunter, W. G., and J. S. Hunter. Statistics for Experimenters. Wiley. New York. 1978.

Walpole, R. E. and R. H. Myers. Probability and Statistics for Engineers and Scientists. Prentice-Hall. <http://www.elcom-hu.com/Mshtrk/Statstics/9th%20txt%20book.pdf>

Hayter, A. Probability and Statistics for Engineers and Scientists. Brooks/Cole. https://zackrauen.com/PublicFiles/School/Textbooks/STAT383_Stats.pdf

Romeu, J. L. Operations Research and Statistics Techniques. Proceedings of Federal Conference on Statistical Methodology. <https://web.cortland.edu/matresearch/OR&StatsFCSMPaper.pdf>

Romeu, J. L. Statistical Assumptions of an Exponential Distribution. RAC START. Vol. 8, No. 2 <https://web.cortland.edu/matresearch/ExpAssumSTART.pdf>

Romeu, J. L. Empirical Assessment of Normal and Lognormal Distributions. RAC START. Vol. 9, No. 6. <https://web.cortland.edu/matresearch/NormAssumSTART.pdf>

About the Author:

Jorge Luis Romeu retired Emeritus from the State University of New York (SUNY). He was, for sixteen years, a Research Professor at Syracuse University, where he is currently an Adjunct Professor of Statistics. Romeu worked for many years as a Senior Research Engineer with the Reliability Analysis Center (RAC), an Air Force Information and Analysis Center operated by IIT Research Institute (IITRI). Romeu received seven Fulbright assignments: in Mexico (3), the Dominican Republic (2), Ecuador, and Colombia. He holds a doctorate in Statistics/O.R., is a C. Stat. Fellow, of the Royal Statistical Society, a Senior Member of the American Society for Quality (ASQ), and Member of the American Statistical Association. He is a Past ASQ Regional Director (and currently a Deputy Regional Director), and holds Reliability and Quality ASQ Professional Certifications. Romeu created and directs the Juarez Lincoln Marti International Ed. Project (JLM, <https://web.cortland.edu/matresearch/>), which supports (i) higher education in Ibero-America and (ii) maintains the Quality, Reliability and Continuous Improvement Institute (QR&CII, <https://web.cortland.edu/romeu/QR&CII.htm>) applied statistics web site.