Understanding and Using Availability

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Webinar Take-Aways

- Understanding Availability from a practical standpoint
- Calculating different Availability ratings
- Practical and Economic ways of enhancing Availability

Summary

Availability is a performance measure concerned with assessing a maintained system or device, with respect to its ability to be used when needed. We overview how it is measured under its three different definitions, and via several methods (theoretical/practical), using both statistical and Markov approaches. We overview the cases where redundancy is used and where degradation is allowed. Finally, we discuss ways of improving Availability and provide numerical examples.

When to use Availability

- When system/device can fail and be repaired
  - During “maintenance”, system is “down”
  - After “maintenance”, system is again “up”
- Formal Definition: “a measure of the degree to which an item is in an operable state at any time.” (Reliability Toolkit, RIAC)

System Availability

- A probabilistic concept based on:
- Two Random Variables X and Y
  - X, System or device time between failures
  - Y, Maintenance or repair time
- Long run averages of X and Y are:
  - E(X) Mean time Between Failures (MTBF)
  - E(Y) Expected Maintenance Time (MTTR)

Availability by Mission Type

- **Blanchard** (Ref. 2): **availability** may be expressed differently, depending on the system and its mission. There are three types of Availability:
  - Inherent
  - Achieved
  - Operational

Inherent Availability: A_i

* Probability that a system, when used under stated conditions, will operate satisfactorily at any point in time.

* A_i <u>excludes</u> preventive maintenance, logistics and administrative delays, etc.

$$A_i = \frac{MTBF}{MTBF + MTTR}$$

Achieved Availability: A_a

* Probability that a system, when used under stated conditions, will operate satisfactorily at any point in time, when called upon.
* A_i <u>includes</u> other activities such as preventive maintenance, logistics, etc.

Operational Availability: A_o

* Probability that a system, when used under stated conditions will operate satisfactorily when called upon.
* A_o includes <u>all factors</u> that contribute to system downtime (now called Mean Down Time, MDT) for <u>all</u> reasons (maintenance actions and delays, access, diagnostics, active repair, supply delays, etc.).

A_o Long Run average formula:

$$A_o = P(System.Up) = \frac{Fav.Cases}{Tot.Cases}$$

$$= \frac{Up.Time}{Cycle.Time} = \frac{E(X)}{E(X)+E(Y)}$$

$$= \frac{MTBF}{MTBF + MDT}$$

Numerical Example

Event	**SubEvent**	**Time**	**Inherent**	**Achieved**	**Operational**
Up	Running	50	50	50	50
Down	Wait-D	10			10
Down	Diagnose	5	5	5	5
Down	Wait-S	3			3
Down	Wait-Adm	2			2
Down	Install	8	8	8	8
Down	Wait-Adm	3			3
Up	Running	45	45	45	45
Down	Preventive	7		7	7
Up	Running	52	52	52	52

UpTime	147	147	147	147
Maintenance		13	20	38
Availability		**0.9188**	**0.8802**	**0.7946**

Formal Definition of Availability

Hoyland et al (Ref. 1): <u>*availability at time* t</u>, denoted A(t), is *the probability that the system is functioning (up and running) at time t*.

X(t): the <u>*state*</u> of a system at time "t"

* "up" and running, [X(t) = 1],
* "down" and failed [X(t) = 0]

A(t) can then be written:

A(t) = P{X(T) = 1}; t > 0

Availability as a R. V.

$$A = \frac{X}{X+Y}; X, Y > 0$$

- The problem of obtaining the “density function” of A
  - resolved via variable transformation of the joint distribution
- Based on the two Random Variables X and Y
  - time to failure X, and time to repair Y
- Expected and Variance of the Availability r.v.
- L_{10} (10th Percentile of A) = P{A < 0.1} = 0.1
  - First and Third Quartiles of Availability, etc.
- Theoretical results, approximated by Monte Carlo

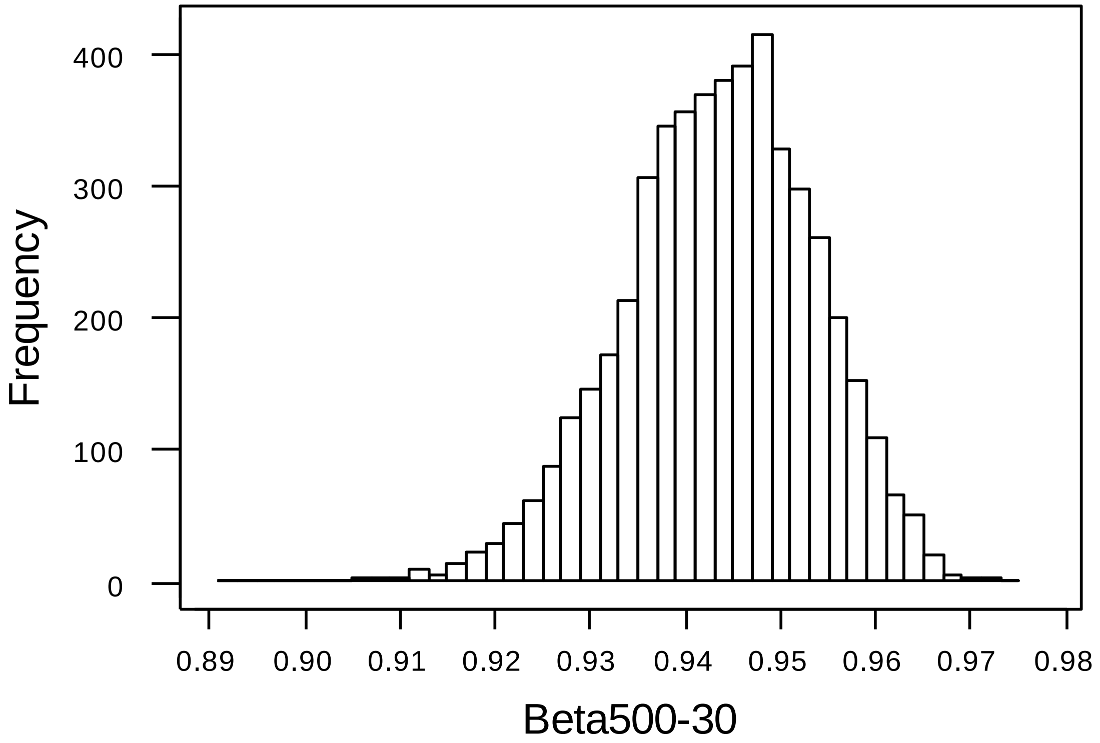
Monte Carlo Simulation

- Generate n = 5000 random Exponential failure and repair times: X_i and Y_i
- Obtain the corresponding Availabilities: $A_i = X_i /(X_i + Y_i); 1 \leq i \leq 5000$
- Sort them, and calculate all the n = 5000 A_i results, numerically
- Obtain the desired parameters from them.

Numerical Example

- Use Beta distribution for expediency
  - Ratio yielding A_i is distributed Beta(μ_1; μ_2)
- Time to failure (X) mean: $\mu_1 = 500$ hours
- Time to repair (Y): $\mu_2 = 30$ hours
- Generate n = 5000 random Beta values
  - with the above parameters μ_1 and μ_2
- Obtain the MC Availabilities: A_i

Histogram of Example



Estimated Parameters of Example

MC Results for Beta(500,30) Example:
Average Availability = 0.9435
Variance of Availability = 9.92×10^{-5}
Life L_{10} = 0.9305
Quartiles: 0.9370 and 0.9505
P(A) > 0.95 $\underline{\approx}$ 0.9499 → $R_{0.949}$=3653

$$P\{A > 0.95\} = 1 - P\{A \le 0.95\} \approx 1 - \frac{3653}{5000} = 1 - 0.7306 = 0.2694$$

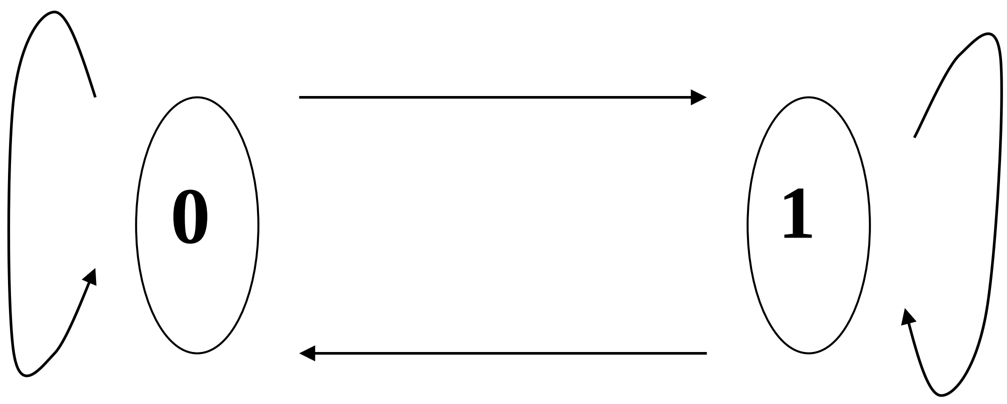
Markov Model Approach

- Two-state Markov Chain (Refs. 4, 5, 6, 7)
- Monitor *status* of system at time T: X(T)
- Denote State 0 (Down), and State 1 (Up)
- X(T) = 0: system S is down at time T
- What is the probability q (or p) that system S is Up (or Down) at time T, given that it was Down (or Up) at time T-1?

Markov Representation of S:

$$p_{01} = P\{X(T) = 1 | X(T-1) = 0\} = q$$

$$p_{10} = P\{X(T) = 0 \mid X(T-1) = 1\} = p$$



Numerical Example:

- System S is in state Up; then moves to state Down in one step, with Prob. $p_{10} = p = 0.002$
  - A Geometric distribution with Mean $\mu = 1/p = 500$ hours.
- System S is in state Down; then moves to state Up in one-step, with Probability $p_{01} = q = 0.033$
  - A Geometric distribution, with Mean $\mu = 1/q = 30$ hours.
- Every step (time period to transition) is an hour.
- The Geometric distribution is the Discrete counterpart of the Continuous Exponential

Transition Probability Matrix P

$$\begin{array}{ccc} States & 0 & 1 \\ 0 & (1-q & q) \\ 1 & (p & 1-p) \end{array} = \begin{array}{ccc} States & 0 & 1 \\ 0 & (0.967 & 0.033) \\ 1 & (0.002 & 0.998) \end{array}$$

Entries of Matrix P = (p_{ij}) correspond to the Markov Chain's one-step transition probabilities. Rows represent every system state that S can be in, at time T. Columns represent every other state that S can go into, in one step (i.e. where S will be, at time T+1).

Obtain the probability of S moving from state Up to Down, in Two Hours

$$P^2 = \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix}^2 = \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix} \times \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix}$$

$$= \begin{bmatrix} (1-q)^2 + pq & q(1-q) + q(1-p) \\ p(1-q) + p(1-p) & pq + (1-p)^2 \end{bmatrix} = \begin{bmatrix} p_{oo}^{(2)} & p_{01}^{(2)} \\ p_{10}^{(2)} & p_{11}^{(2)} \end{bmatrix}$$

$$\Rightarrow p_{10}^{(2)} = p_{10}p_{00} + p_{11}p_{10} = p(1-q) + (1-p)p$$

Hence, the probability S will go down in two hours is:

$$p_{10}^{(2)} = p(1-q) + (1-p)p = 0.003$$

Other useful Markov results:

- If $p_{10}^{(2)} = 0.003 \Rightarrow p_{11}^{(2)} = 1 - p_{10}^{(2)} = A(T) = 0.993$
  – system Availability, after T=2 hours of operation
- Prob. of moving from state 1 to 0, in 10 steps:
  – $(P)^{10} \Rightarrow p_{10}^{(10)} = 0.017$; includes that S could have gone Down or Up, then restored again, several times.
- For sufficiently large **n** (long run) and two-states:

$$Limit_{n\to\infty} P^n = Limit_{n\to\infty} \{ \frac{1}{p+q} \begin{bmatrix} p & q \\ p & q \end{bmatrix} + \frac{(1-p-q)^n}{p+q} \begin{bmatrix} q & -q \\ -p & p \end{bmatrix} \} = \begin{bmatrix} p/(p+q) & q/(p+q) \\ p/(p+q) & q/(p+q) \end{bmatrix}$$

Example: Up = q/(p+q) = 0.943 ; Down = p/(p+q) = 0.057

Markov Model for redundant system

- A Redundant System is composed of two identical devices, in parallel.
- The System is maintained and can function at a degraded level, with only one unit UP.
- The System has now three States: 0, 1, 2:
  - State 0, the <u>Down state</u>; both units are DOWN
  - State 1, the <u>Degraded state</u>; only one unit is UP
  - State 2, the <u>UP state</u>; both units are operating

Markov Model system representation

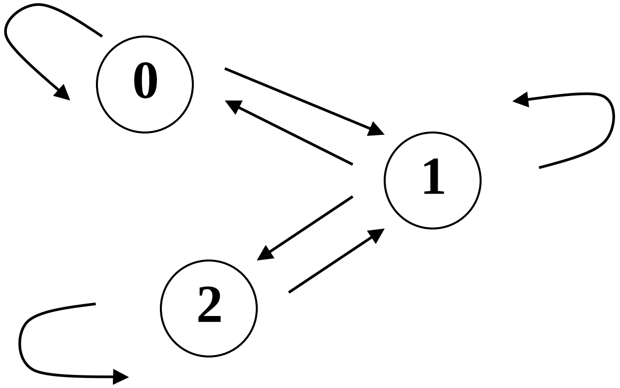
$$p_{01} = P\{X(T) = 1 \mid X(T-1) = 0\} = q$$

$$p_{10} = P\{X(T) = 0 \mid X(T-1) = 1\} = p$$

$$p_{12} = P\{X(T) = 2 \mid X(T-1) = 1\} = q$$

$$p_{21} = P\{X(T) = 1 \mid X(T-1) = 2\} = 2p$$

$$p_{ii} = P\{X(T) = i \mid X(T-1) = i\} = 1 - \sum_{j \neq i} p_{ij}$$



Operational Conditions

- Every step (hour) T is an independent trial
- Success Prob. p_{ij} corresponds to a transition from current state ‘i’ into state ‘j’ = 0,1,2
- Distribution of every change of state is the Geometric (Counterpart of the Exponential)
- Mean time to accomplishing such change of state is: $\mu = 1/p_{ij}$

Transition Probability Matrix P:

$$P = \begin{array}{cccc} States & 0 & 1 & 2 \\ 0 & p_{00} & p_{01} & p_{02} \\ 1 & p_{10} & p_{11} & p_{12} \\ 2 & p_{20} & p_{21} & p_{22} \end{array} = \begin{array}{cccc} States & 0 & 1 & 2 \\ 0 & 1-q & q & 0 \\ 1 & p & 1-p-q & q \\ 2 & 0 & 2p & 1-2p \end{array}$$

As before, the probability of being in state "j" after "n" steps, given that we started in some state "i" of S, is obtained by raising matrix P to the power "n", and then looking at entry p_{ij} *of the resulting matrix* P^n.

Numerical Example

- Probability p of either unit failing
  - in the next hour is 0.002
- Probability q of the repair crew completing
  - a maintenance job in the next hour is 0.033
- Only one failure is allowed
  - in each unit time period,
- and only one repair can be undertaken
  - at any unit time

Probability that a degraded system (in State 1) remains degraded after two hours of operation:

- Sum probabilities corresponding to 3 events
  - the system status has never changed.
  - one unit repaired but another fails during 2^{nd} hour
  - remaining unit fails in the first hour (system goes down), but a repair is completed in the 2^{nd} hour

Numerical Example:

$$P_{11}^{2} = [P \times P]_{11} = p_{11}^{(2)} = p_{10}p_{01} + p_{11}p_{11} + p_{12}p_{21}$$

$$= pq + (1 - p - q)^{2} + 2pq$$

$$= 0.002 \times 0.033 + (1 - 0.035)^{2} + 2 \times 0.002 \times 0.033$$

$$P_{11}^{2} = 0.9314$$

The probability that a system, in degraded state, is still in degraded state after two hours, is: $P^{2}_{11} = 0.9314$

Mean time μ that the system S spends in the Degraded state

- System S can change to Up or Down
  - with probabilities p and q, respectively
- S will remain in the state Degraded
  - with probability 1- p- q (i.e. no change)
- On average, S will spend a "sojourn" of
  - length $1/(p + q) = 1/0.035 = 28.57$ hrs
  - in the Degraded state, before moving out.

Availability at time T

- A(T) = P{S is Available at T}
- System S is not Down at time “T
  - Then, S can be either Up, or Degraded
- A(T) depends on the initial state of S
- Find Prob. S is “Degraded Available” at T,
  - given that S was Degraded at T=0 (initially).

$$p_{10}^{(T)} + p_{11}^{(T)} + p_{12}^{(T)} = 1 \Rightarrow p_{11}^{(T)} = 1 - p_{10}^{(T)} - p_{12}^{(T)}$$

$$A(T) = P\{X(T) = 1 \mid X(0) = 1\} = p_{11}^{(T)} = 1 - p_{10}^{(T)} - p_{12}^{(T)}$$

State Occupancies

- Long run averages of system sojourns
- Asymptotic probabilities of system S being
  – in each one of its possible states at any time T
- Or the percent time S spent in these states
  – Irrespective of the state S was in, initially.
- Results are obtained by considering
  – Vector Π of the "long run" probabilities:

Characteristics of Vector Π

$$\Pi = Limit_{T\to\infty}(\Pr ob\{X(T)=0\},\Pr ob\{X(T)=1\},\Pr ob\{X(T)=2\})$$

Vector Π fulfills two important properties:

$$(1):\Pi\times P=\Pi;(2)\sum\Pi_i=1; with:\Pi_i=Limit_{T\to\infty}\Pr ob\{X(T)=i\}$$

Π × P = Π (Vector Π times matrix P equals Π) defines a system of linear equations, “normalized” by the 2nd property.

$$\Pi\times P=\left(\Pi_0,\Pi_1,\Pi_2\right)\times\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}=\left(\Pi_0,\Pi_1,\Pi_2\right)$$

$$with:\sum_i\Pi_i=\Pi_0+\Pi_1+\Pi_2=1$$

Numerical Example

$$\Pi \times P = \left(\Pi_0, \Pi_1, \Pi_2\right) \times \begin{bmatrix} 0.967 & 0.033 & 0 \\ 0.002 & 0.965 & 0.033 \\ 0 & 0.004 & 0.996 \end{bmatrix} = \left(\Pi_0, \Pi_1, \Pi_2\right)$$

$$\Rightarrow \begin{cases} 0.967\Pi_0 + 0.002\Pi_1 = \Pi_0 \\ 0.033\Pi_0 + 0.965\Pi_1 + 0.004\Pi_2 = \Pi_1; with: \sum_i \Pi_i = \Pi_0 + \Pi_1 + \Pi_2 = 1 \\ 0.033\Pi_1 + 0.996\Pi_2 = \Pi_2 \end{cases}$$

Solution of the system yields long run occupancy rates:

$$\Pi = \left(\Pi_0, \Pi_1, \Pi_2\right) = \left(0.0065, 0.1074, 0.8861\right)$$

Interpretation of results:

- $\Pi_2 = 0.8861$ indicates that system S
  - is operating at full capacity 88% of the time.
- $\Pi_1 = 0.1074$ indicates that system S
  - is operating at Degraded capacity 10% of the time.
- Π_0: probability corresponding to State 0 (Down)
  - is associated with S being Unavailable (= 0.0065)
- "long run" System Availability is given by:
  - $A = 1 - \Pi_0 = 1 - 0.0065 = 0.9935$

Expected Times

- For System S to go Down, if initially
  - S was Degraded (denoted V_1), or Up (V_2)
- Or the average time System S spent in each
  - of these states (1, 2) before going "Down".
- Assume Down is an "absorbing" state
  - one that, once entered, can never be left
- Solve a system of equations leading to
  - all such possible situations.

Numerical Example:

One step, at minimum (initial visit), before system S goes Down. If S is not absorbed then, system S will move on to any of other, non-absorbing (Up, Degraded) state with corresponding probability, and then the process restarts:

$$V_1 = 1 + p_{11}V_1 + p_{12}V_2 = 1 + 0.965V_1 + 0.033V_2$$

$$V_2 = 1 + p_{21}V_1 + p_{22}V_2 = 1 + 0.004V_1 + 0.996V_2$$

Average times until system S goes down yield:
V_1= 4625 (if starting in state Degraded) and
V_2 = 4875 (if starting in state Up).

Model Comparisons

- The initially non-maintained system version,
  - would work an Expected $3/2\lambda = 3/0.004 = 750$
  - hours in Up state, before going Down (Ref. 7).
- The fact that maintenance is now possible, and that S can operate in a Degraded state:
  - results in an increase of $\mu/2\lambda^2 = 0.033/2\times 0.002^2 = 4125$
  - hours in its Expected Time to go Down (from Up).
- The improved Expected Time is due to the Sum of the Two Expected times to failures:
  - $V_2 = 3/2\lambda + \mu/2\lambda^2 = 750 + 4125 = 4875$

Conclusions

- Availability is the ratio of:
  - *Up.Time* to *Cycle.Time*
- Hence, we can enhance Availability by:
  - Increasing the device or system Life
  - Decreasing the maintenance time
  - Simultaneously, doing both above.
- Decreasing maintenance is usually easier.

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