Group 4 Jessica Sass, Kellie Martin, Tyler Fleming ✓

Chapter 7 #5

Max $2x_1 + 3x_2$

S.t.

$4x_1 + 9x_2 \leq 36$

$7x_1 + 5x_2 \leq 35$

$x_1, x_2 \geq 0$ and x_1 integer

Graphic Solution:

$4x_1 + 9x_2 \leq 36$

$\frac{9x_2}{9} = \frac{-4x_1 + 36}{9}$

$x_2 = -\frac{4}{9}x_1 + 4$

$x_2 = 4$ when $x_1 = 0$

$0 = -\frac{4}{9}x_1 + 4$

$-4 = -\frac{4}{9}x_1$

$x_1 = 9$ when $x_2 = 0$

$7x_1 + 5x_2 \leq 35$

$\frac{5x_2}{5} \leq \frac{-7x_1 + 35}{5}$

$x_2 = -\frac{7}{5}x_1 + 7$

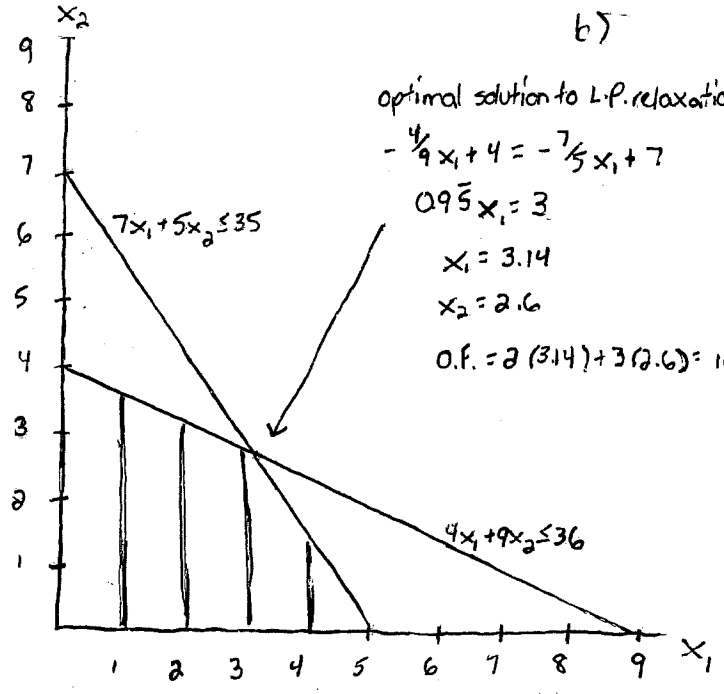
$x_2 = 7$ when $x_1 = 0$

$0 = -\frac{7}{5}x_1 + 7$

$-7 = -\frac{7}{5}x_1$

$x_1 = 5$ when $x_2 = 0$

a)



b)

optimal solution to L.P. relaxation:

$-\frac{4}{9}x_1 + 4 = -\frac{7}{5}x_1 + 7$

$0.95x_1 = 3$

$x_1 = 3.14$

$x_2 = 2.6$

O.F. $= 2(3.14) + 3(2.6) = 14.08$

round x_1 down to 3

O.F. max $2x_1+3x_2$

$2(3)+3(2.6)$

$= 13.8$

not optimal because with $x_1 = 3$ x_2 can still be made larger without violating the constraints

c) see Solver: optimal solution is $x_1 = 3$, $x_2 = 2.67$

$2(3) + 3(2.67) = 14$

Branch and Bound Technique

solution of relaxed (standard) linear program:

objective value = 14.09302

$x_1 = 3.139535$

$x_2 = 2.604651$

solution of 14.09302 is an upper bound for the integer solution

only variable x_1 needs to be an integer so take $x_1 = 3.139535$

and break it into $x_1 \leq 3$ and $x_1 \geq 4$

solution for branch $x_1 \leq 3$:

O.F. = 14

$x_1 = 3$

$x_2 = 2.67$

solution for branch $x_1 \geq 4$

O.F. = 12.2

$x_1 = 4$

$x_2 = 1.4$

optimal solution because O.F. is greater

Group 4: Jessica Sass, Kellie Martin, Tyler Fleming 11/12/2012
Chapter 7 #5

Integer Linear Programming

	x1	x2	
value	3	2.666667	
O.F.	maximize 2x1 + 3x2		
	14		
S.T.	4x1 + 9x2	<=	36
constraints:	7x1 + 5x2	<=	35
	x1	>=	0
	x2	>=	0
	x1	=	integer
	36	<=	36
	34.33333	<=	35
	3	>=	0
	2.666667	>=	0
	3	=	integer

Microsoft Excel 12.0 Answer Report
Worksheet: [excel hw solution.xlsx]Sheet1
Report Created: 11/12/2012 3:05:44 PM

Target Cell (Max)

Cell	Name	Original Value	Final Value
$B$10	maximize 2x1 + 3x2	14	14

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$7	value x1	3	3
$C$7	value x2	2.666666667	2.666666667

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$18	x1	36	$B$18<=$D$18	Binding	0
$B$19	x1	34.33333333	$B$19<=$D$19	Not Binding	0.666666667
$B$20	x1	3	$B$20>=$D$20	Not Binding	3
$B$21	x1	2.666666667	$B$21>=$D$21	Not Binding	2.666666667
$B$7	value x1	3	$B$7=integer	Binding	0

Branch and Bound Technique

Solution of Relaxed (Standard) Linear Program

	x1	x2	
value	3.139535	2.604651	
O.F.	maximize 2x1 + 3x2		
	14.09302		
S.T.	4x1 + 9x2	<=	36
constraints	7x1 + 5x2	<=	35
	x1	>=	0
	x2	>=	0
	36	<=	36
	35	<=	35
	3.139535	>=	0
	2.604651	>=	0

Microsoft Excel 14.0 Answer Report
Worksheet: [excel hw solution.xlsx]Sheet2
Report Created: 11/13/2012 1:33:39 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine

Engine: GRG Nonlinear
Solution Time: 0.016 Seconds.
Iterations: 0 Subproblems: 0

Solver Options

Max Time 100 sec, Iterations 100, Precision 0.000001
Convergence 0.0001, Population Size 100, Random Seed 0, Derivatives Forward, Require Bounds
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 5%, Solve Without Integer Co

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$B$9	maximize 2x1 + 3x2	14.09302326	14.09302326

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$6	value x1	3.139534884	3.139534884	Contin
$C$6	value x2	2.604651163	2.604651163	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$16	x2	36	$B$16<=$D$16	Binding	0
$B$17	x2	35	$B$17<=$D$17	Binding	0
$B$18	x2	3.139534884	$B$18>=$D$18	Not Binding	3.139534884
$B$19	x2	2.604651163	$B$19>=$D$19	Not Binding	2.604651163

Microsoft Excel 14.0 Sensitivity Report
Worksheet: [excel hw solution.xlsx]Sheet2
Report Created: 11/13/2012 1:33:39 PM

Variable Cells

Cell	Name	Final Value	Reduced Gradient
$B$6	value x1	3.139534884	0
$C$6	value x2	2.604651163	0

Constraints

Cell	Name	Final Value	Lagrange Multiplier
$B$16	x2	36	0.255813953
$B$17	x2	35	0.139534884
$B$18	x2	3.139534884	0
$B$19	x2	2.604651163	0

Solution with branch x1<= 3

	x1	x2
value	3	2.666667

O.F. maximize 2x1 + 3x2

14

S.T.	4x1 + 9x2	<=	36
constraints	7x1 + 5x2	<=	35
	x1	>=	0
	x2	>=	0
	x1	<=	3
	36	<=	36
	34.33333	<=	35
	3	>=	0
	2.666667	>=	0
	3	<=	3

Microsoft Excel 14.0 Answer Report
Worksheet: [excel hw solution.xlsx]Sheet3
Report Created: 11/13/2012 1:35:09 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine

Engine: GRG Nonlinear
Solution Time: 0 Seconds.
Iterations: 0 Subproblems: 0

Solver Options

Max Time 100 sec, Iterations 100, Precision 0.000001
Convergence 0.0001, Population Size 100, Random Seed 0, Derivatives Forward, Require Bounds
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 5%, Solve Without Integer Co

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$B$7	maximize 2x1 + 3x2	14	14

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$4	value x1	3	3	Contin
$C$4	value x2	2.666666667	2.666666667	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$15	x1	36	$B$15<=$D$15	Binding	0
$B$16	x1	34.33333333	$B$16<=$D$16	Not Binding	0.666666667
$B$17	x1	3	$B$17>=$D$17	Not Binding	3
$B$18	x1	2.666666667	$B$18>=$D$18	Not Binding	2.666666667
$B$19	x1	3	$B$19<=$D$19	Binding	0

Microsoft Excel 14.0 Sensitivity Report
Worksheet: [excel hw solution.xlsx]Sheet3
Report Created: 11/13/2012 1:35:10 PM

Variable Cells

Cell	Name	Final Value	Reduced Gradient
$B$4	value x1	3	0
$C$4	value x2	2.666666667	0

Constraints

Cell	Name	Final Value	Lagrange Multiplier
$B$15	x1	36	0.333333333
$B$16	x1	34.33333333	0
$B$17	x1	3	0
$B$18	x1	2.666666667	0
$B$19	x1	3	0.666666667

Solution with branch x1 >= 4

	x1	x2	
value	4	1.4	
O.F.	maximize 2x1 + 3x2		
	12.2		
S.T.	4x1 + 9x2	<=	36
constraints	7x1 + 5x2	<=	35
	x1	>=	0
	x2	>=	0
	x1	>=	4
	28.6	<=	36
	35	<=	35
	4	>=	0
	1.4	>=	0
	4	>=	4

Microsoft Excel 14.0 Answer Report
Worksheet: [excel hw solution.xlsx]Sheet4
Report Created: 11/13/2012 1:36:24 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine

Engine: GRG Nonlinear
Solution Time: 0 Seconds.
Iterations: 0 Subproblems: 0

Solver Options

Max Time 100 sec, Iterations 100, Precision 0.000001
Convergence 0.0001, Population Size 100, Random Seed 0, Derivatives Forward, Require Bounds
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 5%, Solve Without Integer Constı

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$B$7	maximize 2x1 + 3x2	12.2	12.2

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$4	value x1	4	4	Contin
$C$4	value x2	1.4	1.4	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$15	x1	28.6	$B$15<=$D$15	Not Binding	7.4
$B$16	x1	35	$B$16<=$D$16	Binding	0
$B$17	x1	4	$B$17>=$D$17	Not Binding	4
$B$18	x1	1.4	$B$18>=$D$18	Not Binding	1.4
$B$19	x1	4	$B$19>=$D$19	Binding	0

Microsoft Excel 14.0 Sensitivity Report
Worksheet: [excel hw solution.xlsx]Sheet4
Report Created: 11/13/2012 1:36:25 PM

Variable Cells

Cell	Name	Final Value	Reduced Gradient
$B$4	value x1	4	0
$C$4	value x2	1.4	0

Constraints

Cell	Name	Final Value	Lagrange Multiplier
$B$15	x1	28.6	0
$B$16	x1	35	0.6
$B$17	x1	4	0
$B$18	x1	1.4	0
$B$19	x1	4	-2.2