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POWERFUL TECHNIQUES IN RMQSI FROM THE RIAC

UNDERSTANDING LOGISTICS IN SYSTEM ANALYSIS

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INTRODUCTION

The possibility of providing maintenance to a system, thereby extending its life and usefulness, introduces two important concepts into systems analysis: availability and logistics. Availability is the probability that "an item is in an operable and committable state at the start of a mission, when the mission is called for at an unknown (random) time" (1), and has been discussed at length in another paper (2). Related to availability, maintainability is "a measure of the ability of an item to be retained in, or restored to, a specified condition when the maintenance is performed during the course of a specified mission profile" (1). Maintainability, in turn, brings in the topic of this article: logistics. As defined in (3), integrated logistics support (ILS) is a "disciplined, unified, and iterative approach to the management and technical activities necessary to (i) integrate support considerations into system and equipment design; (ii) develop support requirements related consistently to readiness objectives, to design, and to each other; (iii) acquire the required support; and (iv) provide the required support during the operational phase at minimum cost."

As seen in the above definition, logistics includes not only the maintenance itself (e.g., repairs) but also the organization behind all such maintenance operations as well as parts and equipment necessary to perform repairs during the system's operational phase. It is further desirable to achieve logistics support at a minimum cost. Therefore, we need to consider, in addition to the administrative and organizational issues related to its management, the statistical models required to analyze the random processes behind system breakdowns and repairs.

This article provides an example of the advantages of including maintenance as part of a system's logistic strategy, and then overviews several statistics techniques that can help plan and optimize the inventory required in the logistics activity.

ADVANTAGES OF SYSTEM MAINTENANCE

Systems can be of two types: "one shot," if they operate until they fail, or "maintained," if they can be repaired and put back into service when they go down. Comparing the two approaches for equivalent systems can assess the advantages of each. For example, a one shot system is presented in Figure 1-a. It is composed of two identical units in parallel such that the system can be in three states: up (both units operating), down (neither unit operating) and degraded (one of two units operating). On the other hand, if the system is maintained, provision is made for repairing the system when it goes down. A maintained version of our above example would have the same configuration, but now failed units may be repaired and returned to operation. This considerably extends system life. The system also works in a degraded state while being repaired, as long as there is one unit still functioning (Figure 1-b).

Figure 1. Parallel system of two identical units: (a) block diagram; (b) state diagram.

As an illustration, let each of the two identical components have a mean time to failure (MTTF) of MTTF = $\mu = 1/\lambda = 500$ hours ($\lambda = 0.002$ is the rate parameter for an exponential probability density function describing time to failure). Then, the long-run total system MTTF is (4):

$$MTTF = \mu = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1 + \lambda_2} = \frac{2}{\lambda} - \frac{1}{2\lambda} = \frac{3}{2} \times 500 = 750 \text{ hours} .$$

Hence, the non-maintained version would have 750 hours of expected life before going down.

For comparison, suppose the system is maintained, with mean time to repair (MTTR) $\eta = 30$ hours ($\rho = 1/\eta = 0.033$), when operating in a degraded state, as in Figure 1b. Since maintenance is now possible, the system achieves an increase of its expected life (mean time to go down) of $\rho/2\lambda^2 = 0.033/(2x0.002^2) = 4125$ hours. Such an increase occurs because the new expected life of the system is now the sum of the original non-maintained expected time to failure, plus the additional expected time produced by having system maintenance ($\rho/2\lambda^2$). Consequently, the new overall expected time for the system to go down (mean time between failures [MTBF], since the system can now be repaired), starting from the state of being up, becomes:

$$\text{MTBF} = 3/2\lambda + \rho/2\lambda^2 = 750 + 4125 = 4875.$$

System maintenance also allows the system to achieve, during its life, an availability (A) of:

$$A = P\{\text{system is up}\} = \frac{\text{up time}}{\text{cycle time}} = \frac{MTBF}{MTBF + MTTR} = \frac{4875}{4875+30} = 0.9938.$$

This example illustrates some of the practical advantages of maintainable systems. It also underlines how a reduction in system maintenance times MTTR produces an increase in system availability, among other benefits to the entire long-term systems operation. Maintenance time reduction may be obtained by having an efficient logistics program in place. In the rest of this paper, the concept of logistics and several ways to improve logistics-related activities, from a managerial as well as from a statistical standpoint, are discussed.

SOME STATISTICAL PROCEDURES USED IN LOGISTICS

Improvements in the system maintenance process can extend system life and availability. But maintenance time includes, in addition to repairs proper, the logistics delay time (i.e., waiting for parts if they are not in stock). Logistics planning requires the calculation of the adequate number of parts that we must keep in inventory (sparing) so as to reduce the risk of system unavailability and down time. In this section I overview some of the statistical issues in logistics planning.

1. Sparing models: determining the number of spare parts.

All sparing derivations below correspond to exponential lifetimes only. This implies that the failure rate is time independent (constant), and that the number of failures observed in time interval [0, t) are Poisson-distributed.

Suppose a system requires one operating part to fulfill its mission in time interval [0, t). How many spares, k, of such part, are needed in the inventory to avoid logistics delay? We first need to define the reliability requirements, $R(t)$, of the system in order for us to keep enough spares, k, to immediately replace the number of failed parts during mission time, t. Due to the exponential lifetime assumption, the number of failures during mission time t is a Poisson random variable:

$R(t) = P\{\text{system successfully completes mission}\} = P\{\#\text{ of failures in } [0, t) \text{ is} \le k\} = P\{N(t) \le k\}$

$$= \sum_{n=0}^{k} P\{N(t)=n\} = P\{N(t)=0\} + \ldots + P(N(t)=k\} = e^{-\lambda t} + \ldots + e^{-\lambda t}(\lambda t)^k / k! \;.$$

To find the number of spares required for immediate repair, the equation is solved for k, and desired reliability, $R(t)$.

Example: Let failure rate $\lambda = 0.01$/hr and mission time $t = 1000$ hours. Find the number of spares k such that system reliability is $R(t) > 0.95$. Using Poisson tables for $\lambda t = 10$ we obtain:

$$R(t) = \sum_{j=0}^{k} P\{N(t)=j\} = e^{-\lambda t} + \ldots + \frac{(\lambda t)^k e^{-\lambda t}}{k!} > 0.95 \quad \Rightarrow \quad k = 15 \text{ yields } R(t) = 0.9512.$$

Therefore, we would need to keep a minimum of $k = 15$ spares, to avoid running out of spares during mission time and thus, ensuring that system reliability is at least 0.95.

2. Additional sparing examples.

Suppose now that system reliability is pre-specified. We want to determine the number of spares k, to have in the inventory, in order to ensure that enough are available at least X percent of the time, regardless of the length of mission time. In this case, we can use a revised version of the reliability model. Note that $R(t) = e^{-\lambda t}$ and that $ln\{R(t)\} = ln\{e^{-\lambda t}\} = -\lambda t$. Therefore, denoting P as the desired probability of having a spare part available, when needed (i.e., while we still have some of the k spares), we can rewrite the Poisson equation:

$$P = P\{N(t) \le k\} = \sum_{n=0}^{k} \frac{(\lambda t)^n e^{-\lambda t}}{n!} = e^{-\lambda t} + \ldots + \frac{(\lambda t)^k e^{-\lambda t}}{k!} = \sum_{n=0}^{k} R(t)\left(-\ln\{R(t)\}\right)^n / n!$$

Example: Suppose that system reliability is $R(t) = 0.8$. What inventory size, S, of available parts are needed to ensure that a spare is at hand, at least 99% of the time?

$$P = 0.99 \le \sum_{n=0}^{S} R(t)\left(-\ln\{R(t)\}\right)^n / n! = \sum_{n=0}^{S} 0.8\left(-\ln\{0.8\}\right)^n / n! = 0.8 + 0.178 + 0.020 + \ldots$$

Hence, it is enough to keep an inventory of $S = 2$ parts, since the above sum, for $n = 0$ to $n = 2$, fulfills the desired P = 0.998 > 0.99.

This model can also be used for more complex sparing situations. For example, suppose that a system requires 12 identical parts to operate, and that each part has a failure rate of $\lambda = 0.01$ failures/1000 hours. If the system must operate continuously for a year (7×24×52 = 8736 hours), what inventory size, S, do we need to keep, to ensure that at least one part will be available, at least $P = 95\%$ of the time?

To compute the system reliability for the required mission time, we need to take into account that all system parts must be working. Hence, we convert the failure rate to hours, and multiply it by the mission time (t = 8736 hours), and by the number of parts that the system requires to operate (k =12 items). That is:

$$k \times l \times t = 12 \times (0.01/1000\text{hours}) \times (8736 \text{ hours}) = 1.04832.$$

Reliability is then computed as: $R(t) = e^{-k\lambda t} = e^{-1.04832} = 0.35053$.

$$P = 0.95 \le \sum_{n=0}^{S} R(t) \times (-\ln\{R(t)\})^n / n! = \sum_{n=0}^{S} 0.35 \times (-\ln\{0.35\})^n / n! = 0.35 + 0.37 + 0.19 + 0.07$$

The above sum yields 0.98, for S = 3. Hence, it is enough to keep three spare parts in inventory, to ensure that a spare part will be available at least 95% of the time. There are also nomographs available, that provide these results directly without having to calculate the above Poisson probabilities (for examples, see Ch. 2 of Reference 3).

3. Determining risk-dependent maintenance times.

Having the proper number of spares in inventory is only one factor in achieving an efficient logistics process. It is also important to be able to accurately predict maintenance times. Fitting a statistical distribution and computing the probability of observing particular maintenance times most easily accomplishes this. Maintenance times can usually be fitted one of several statistical distributions, e.g. normal, lognormal, or exponential (5, 6). A goodness-of-fit (GoF) test can be used on observed data, to determine which distribution fits most appropriately. Then, given a selected risk (probability that maintenance will be completed by a pre-specified time), we can obtain the corresponding time percentile (e.g., max, min, median etc.) from the fitted distribution and use it to plan our logistics needs.

Example: Suppose that we implemented a GoF test on maintenance time data and found that the time to repair (X) has a log normal distribution (7, 8). If X is lognormal, then $\ln(X)$ is normal. Hence, we can log-transform the repair time data and use the standard normal distribution to compute probabilities. Further suppose that the log-transformed maintenance time data yielded a mean of 3.5 and a standard deviation of 1.2, and that we require the 90th percentile of maintenance times (i.e., 90% of all maintenance operations would be completed by such time). This percentile defines the risk we are willing to run in our planning processes (as 10% of the times, maintenance will exceed the planned time).

The standardized 90th percentile (z = 1.28) is obtained from the standard normal table. The back-transformed percentile (Perc.90) of the log-transformed (normal) repair time data is then:

$$90^{th}\ z\text{-percentile} = 1.28 \Rightarrow \frac{Perc.90 - 3.5}{1.2} = 1.28 \Rightarrow Perc.90 = 3.5 + 1.28 \times 1.2 = 5.036\ .$$

Perc.90 = 5.036 corresponds to the log-transformed data; hence, the estimated 90th percentile of the actual maintenance times is $e^{5.036}$ = 153.8 hours. Therefore, 10% of repair times will take more than 153.8 hours to complete. We can use this time (153.8 hours) for logistics planning, with a risk of 10% of overshooting. If the risk required is different (e.g. 5%), solve for another percentile (95th z-percentile = 1.65). If the maintenance time distribution is other than normal or log normal, look up the percentiles in the table of the identified statistical distribution.

4. Other models used in logistics.

There are many operations research (OR) models that can be used to optimize logistics processes. For example, inventory models help determine the organization of sparing policies (e.g. inventory size, reordering times, quantities, etc.). Transportation models help determine the most efficient locations of warehouses. Assignment models help identify the best trucking routes to restock them and which warehouse or job shop should serve which customer. OR models often use linear and integer programming to find an optimal (maximum or minimum) solution of an objective function (performance measure) when it is subjected to a set of constraints (limitations). Goal programming is used to determine the optimal policy in the presence of several competing and conflicting objectives, all of which we want to meet simultaneously.

Details of operations research models are beyond the scope of this article. The interested reader can find extensive discussions of these models, as well as many illustrative examples in (9).

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