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Romer

Outsourcing vs. Manufacturing.

The following example, from the textbook of Operations management by Sweeney et al., illustrates the models used by some industries to determine when to manufacture in-house, or to buy (outsource) a product:

4.3 OPERATIONS MANAGEMENT APPLICATIONS

Linear programming applications developed for production and operations management include scheduling, staffing, inventory control, and capacity planning. In this section we describe examples with make-or-buy decisions, production scheduling, and workforce assignments.

A Make-or-Buy Decision

We illustrate the use of a linear programming model to determine how much of each of several component parts a company should manufacture and how much it should purchase from an outside supplier. Such a decision is referred to as a make-or-buy decision.

The Janders Company markets various business and engineering products. Currently, Janders is preparing to introduce two new calculators: one for the business market called the Financial Manager and one for the engineering market called the Technician. Each calculator has three components: a base, an electronic cartridge, and a faceplate or top. The same base is used for both calculators, but the cartridges and tops are different. All components can be manufactured by the company or purchased from outside suppliers. The manufacturing costs and purchase prices for the components are summarized in Table 4.5.

TABLE 4.5 MANUFACTURING COSTS AND PURCHASE PRICES FOR JANDERS CALCULATOR COMPONENTS

Component	Cost per Unit Manufacture (regular time)	Purchase
Base	$0.50	$0.60
Financial cartridge	$3.75	$4.00
Technician cartridge	$3.30	$3.90
Financial top	$0.60	$0.65
Technician top	$0.75	$0.78

Company forecasters indicate that 3000 Financial Manager calculators and 2000 Technician calculators will be needed. However, manufacturing capacity is limited. The company has 200 hours of regular manufacturing time and 50 hours of overtime that can be scheduled for the calculators. Overtime involves a premium at the additional cost of $9 per hour. Table 4.6 shows manufacturing times (in minutes) for the components.

The problem for Janders is to determine how many units of each component to manufacture and how many units of each component to purchase. We define the decision variables as follows:

(outsource)

BM = number of bases manufactured
BP = number of bases purchased
FCM = number of Financial cartridges manufactured
FCP = number of Financial cartridges purchased
TCM = number of Technician cartridges manufactured
TCP = number of Technician cartridges purchased
FTM = number of Financial tops manufactured
FTP = number of Financial tops purchased
TTM = number of Technician tops manufactured
TTP = number of Technician tops purchased

One additional decision variable is needed to determine the hours of overtime that must be scheduled:

OT = number of hours of overtime to be scheduled

TABLE 4.6 MANUFACTURING TIMES IN MINUTES PER UNIT FOR JANDERS CALCULATOR COMPONENTS

Component	Manufacturing Time
Base	1.0
Financial cartridge	3.0
Technician cartridge	2.5
Financial top	1.0
Technician top	1.5

The objective function is to minimize the total cost, including manufacturing costs, purchase costs, and overtime costs. Using the cost-per-unit data in Table 4.5 and the overtime premium cost rate of \$9 per hour, we write the objective function as

$$\begin{aligned}\text{Min}\quad & 0.5BM + 0.6BP + 3.75FCM + 4FCP + 3.3TCM + 3.9TCP + 0.6FTM \\ & + 0.65FTP + 0.75TTM + 0.78TTP + 9OT\end{aligned}$$

The first five constraints specify the number of each component needed to satisfy the demand for 3000 Financial Manager calculators and 2000 Technician calculators. A total of 5000 base components are needed, with the number of other components depending on the demand for the particular calculator. The five demand constraints are

$$\begin{aligned}BM + BP &= 5000 \quad \text{Bases}\\ FCM + FCP &= 3000 \quad \text{Financial cartridges}\\ TCM + TCP &= 2000 \quad \text{Technician cartridges}\\ FTM + FTP &= 3000 \quad \text{Financial tops}\\ TTM + TTP &= 2000 \quad \text{Technician tops}\end{aligned}$$

Two constraints are needed to guarantee that manufacturing capacities for regular time and overtime cannot be exceeded. The first constraint limits overtime capacity to 50 hours, or

$$OT \leq 50$$

The same units of measure must be used for both the left-hand side and right-hand side of the constraint. In this case, minutes are used.

The second constraint states that the total manufacturing time required for all components must be less than or equal to the total manufacturing capacity, including regular time plus overtime. The manufacturing times for the components are expressed in minutes, so we state the total manufacturing capacity constraint in minutes, with the 200 hours of regular time capacity becoming 60(200) = 12,000 minutes. The actual overtime required is unknown at this point, so we write the overtime as $60OT$ minutes. Using the manufacturing times from Table 4.6, we have

$$BM + 3FCM + 2.5TCM + FTM + 1.5TTM \leq 12{,}000 + 60OT$$

The complete formulation of the Janders make-or-buy problem with all decision variables greater than or equal to zero is

L.P. Model

$$\begin{aligned}\text{Min}\quad & 0.5BM + 0.6BP + 3.75FCM + 4FCP + 3.3TCM + 3.9TCP \\ & + 0.6FTM + 0.65FTP + 0.75TTM + 0.78TTP + 9OT\end{aligned}$$

$$\begin{array}{lrll}
\text{s.t.} & & & \\
 & BM + BP = & 5000 & \text{Bases}\\
 & FCM + FCP = & 3000 & \text{Financial cartridges}\\
 & TCM + TCP = & 2000 & \text{Technician cartridges}\\
 & FTM + FTP = & 3000 & \text{Financial tops}\\
 & TTM + TTP = & 2000 & \text{Technician tops}\\
 & OT \leq & 50 & \text{Overtime hours}\\
 & BM + 3FCM + 2.5TCM + FTM + 1.5TTM \leq & 12{,}000 + 60OT & \text{Manufacturing capacity}
\end{array}$$

The optimal solution to this 11-variable, 7-constraint linear program is shown in Figure 4.5. The optimal solution indicates that all 5000 bases (BM), 667 Financial Manager cartridges (FCM), and 2000 Technician cartridges (TCM) should be manufactured. The remaining 2333 Financial Manager cartridges (FCP), all the Financial Manager tops (FTP),

FIGURE 4.5 THE SOLUTION FOR THE JANDERS MAKE-OR-BUY PROBLEM

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Janders

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Optimal Objective Value =          24443.33333

     Variable             Value            Reduced Cost
  --------------     ---------------    -----------------
        BM              5000.00000             0.00000
        BP                 0.00000             0.01667
        FCM              666.66667             0.00000
        FCP             2333.33333             0.00000
        TCM             2000.00000             0.00000
        TCP                0.00000             0.39167
        FTM                0.00000             0.03333
        FTP             3000.00000             0.00000
        TTM                0.00000             0.09500
        TTP             2000.00000             0.00000
        OT                 0.00000             4.00000

    Constraint          Slack/Surplus         Dual Value
  --------------     ---------------    -----------------
        1                  0.00000             0.58333
        2                  0.00000             4.00000
        3                  0.00000             3.50833
        4                  0.00000             0.65000
        5                  0.00000             0.78000
        6                 50.00000             0.00000
        7                  0.00000            -0.08333

                 Objective        Allowable          Allowable
   Variable      Coefficient      Increase           Decrease
  ----------    ------------     ----------         ----------
      BM           0.50000         0.01667            Infinite
      BP           0.60000         Infinite           0.01667
      FCM          3.75000         0.10000            0.05000
      FCP          4.00000         0.05000            0.10000
      TCM          3.30000         0.39167            Infinite
      TCP          3.90000         Infinite           0.39167
      FTM          0.60000         Infinite           0.03333
      FTP          0.65000         0.03333            Infinite
      TTM          0.75000         Infinite           0.09500
      TTP          0.78000         0.09500            Infinite
      OT           9.00000         Infinite           4.00000

                    RHS            Allowable          Allowable
  Constraint       Value           Increase           Decrease
  ----------    ------------     ----------         ----------
      1          5000.00000      2000.00000         5000.00000
      2          3000.00000        Infinite         2333.33333
      3          2000.00000       800.00000         2000.00000
      4          3000.00000        Infinite         3000.00000
      5          2000.00000        Infinite         2000.00000
      6            50.00000        Infinite           50.00000
      7         12000.00000      7000.00000         2000.00000
```

and all Technician tops (*TTP*) should be purchased. No overtime manufacturing is necessary, and the total cost associated with the optimal make-or-buy plan is $24,443.33.

Sensitivity analysis provides some additional information about the unused overtime capacity. The Reduced Costs column shows that the overtime (*OT*) premium would have to decrease by $4 per hour before overtime production should be considered. That is, if the overtime premium is $9 − $4 = $5 or less, Janders may want to replace some of the purchased components with components manufactured on overtime.

The dual value for the manufacturing capacity constraint 7 is −0.083. This value indicates that an additional hour of manufacturing capacity is worth $0.083 per minute or ($0.083)(60) = $5 per hour. The right-hand-side range for constraint 7 shows that this conclusion is valid until the amount of regular time increases to 19,000 minutes, or 316.7 hours.

Sensitivity analysis also indicates that a change in prices charged by the outside suppliers can affect the optimal solution. For instance, the objective coefficient range for *BP* is 0.583 (0.600 − 0.017) to no upper limit. If the purchase price for bases remains at $0.583 or more, the number of bases purchased (*BP*) will remain at zero. However, if the purchase price drops below $0.583, Janders should begin to purchase rather than manufacture the base component. Similar sensitivity analysis conclusions about the purchase price ranges can be drawn for the other components.

NOTES AND COMMENTS

The proper interpretation of the dual value for manufacturing capacity (constraint 7) in the Janders problem is that an additional hour of manufacturing capacity is worth ($0.083)(60) = $5 per hour. Thus, the company should be willing to pay a premium of $5 per hour over and above the current regular time cost per hour, which is already included in the manufacturing cost of the product. Thus, if the regular time cost is $18 per hour, Janders should be willing to pay up to $18 + $5 = $23 per hour to obtain additional labor capacity.

However, the above calculations do not consider certain expenses that occur when lay-offs, caused by the outsourcing of production, occurs. These include payment of unemployment insurance, loss tax revenues, retraining of unemployed, etc. that are absorbed by government (Society). Next, we analyze such expenses.