MFE634

Reliability Analysis

Group 5

Xiangyu Luo
Ameya Palekar
Krutin Parashuram Patil
Chao Sa

Contents

1. Generate Data and Verify 1

2. Calculate and Analyze the Data 2

2.1 Complete Sample. 2

2.1.1 95% Confidence Interval for the Mean Time to Failure (MTTF) 2

2.1.2 95% Confidence Interval for the Failure Rate (FR) 2

2.1.3 90% the Confidence BOUNDS for MTTF and FR (Hint: Develop 80% CI) 2

2.2. With Truncated at the 7th Failure. 3

2.2.1 95% Confidence Interval for the Mean Time to Failure (MTTF) 3

2.2.2 95% Confidence Interval for the Failure Rate (FR) 4

2.2.3 90% Confidence BOUNDS for MTTF and FR (Hint: Develop 80% CI) 4

2.3. With Truncated at time T= 0.25*MTTF. 5

2.3.1 95% Confidence Interval for the Mean Time to Failure (MTTF) 5

2.3.2 95% Confidence Interval for the Failure Rate (FR) 5

2.3.3 90% Confidence BOUNDS for MTTF and FR (Hint: Develop 80% CI) 5

3. Summary 6

4. Implement 7

4.1 A Fault Tree Analysis for Flood Barrier 7

4.2 A Failure Mode and Effects Analysis for Flood Barrier 8

Appendix A: 9

1. Generate Data and Verify

Generate 25 exponentially distributed times to failure, with MTTF as defined by group number.

Expon. MTTF = 2.5K + GNo*100 = 2.5K + 5 × 100 = 3000

Data Display

```
Failure
1132.1    1758.9       0.2    4442.3     457.7    5120.5    1622.1    2677.4
5358.7     750.4    1913.8    1946.9   11663.9     614.6    1271.3    1602.9
3207.3     599.0     131.8    2425.1    4234.4    4698.1   10460.5     520.5
2478.7
```

Sum of Failure

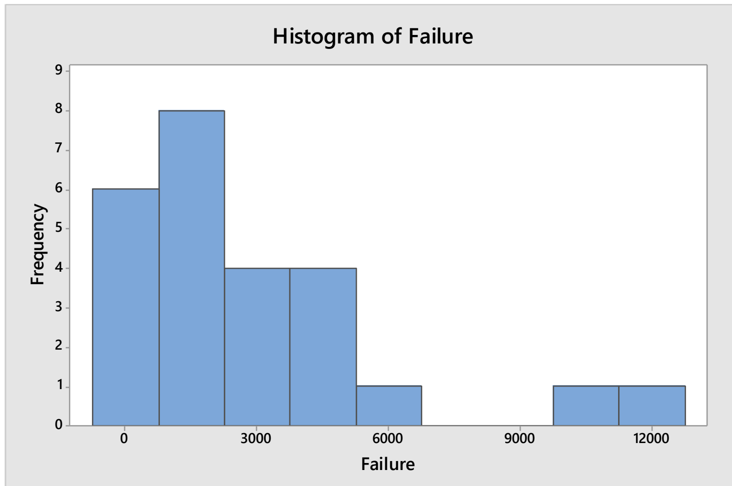
```
Sum of Failure = 71089.1
```

Mean of Failure

```
Mean of Failure = 2843.57
```

Standard Deviation of Failure

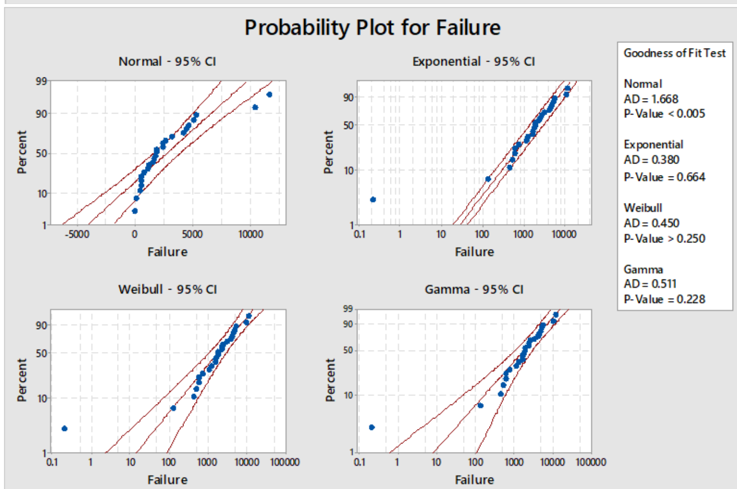
```
Standard deviation of Failure =
2943.33
```



Distribution ID Plot for Failure

```
Goodness of Fit Test

Distribution      AD        P
Normal         1.668   <0.005
Exponential    0.380    0.664
Weibull        0.450   >0.250
Gamma          0.511    0.228
```



Through the highest P-value and lowest AD value, we pick the Exponential distribution as the best fit for our data.

2. Calculate and Analyze the Data

2.1 Complete Sample.

2.1.1 95% Confidence Interval for the Mean Time to Failure (MTTF)

In this case, we have 25 samples and the confidence interval is 95%, and by adding all the test times of failures, we get total time is 71089.

$$n = 25$$

$$\alpha = 0.05$$

$$T = \sum_{i=1}^{25} T_i = 71089$$

With these inputs, we can use the Chi-Squared distribution table to calculate the confidence interval in the experiment with the degree of freedom = 2n:

$$\chi^2_{2n,1-\frac{\alpha}{2}} = \chi^2_{50,0.975} = 71.420$$

$$\chi^2_{2n,\frac{\alpha}{2}} = \chi^2_{50,0.025} = 32.357$$

Hence, the corresponding for the mean life is:

$$\left(\frac{2T}{\chi^2_{2n,1-\frac{\alpha}{2}}}, \frac{2T}{\chi^2_{2n,\frac{\alpha}{2}}}\right) = \left(\frac{142178}{71.420}, \frac{142178}{32.357}\right) = (1990.729, 4394.000)$$

2.1.2 95% Confidence Interval for the Failure Rate (FR)

$$Failure\ Rate = \frac{1}{Mean\ Life}$$

$$= \left(\frac{1}{4394.000}, \frac{1}{1990.729}\right) = (0.0002276, 0.0005023)$$

2.1.3 90% the Confidence BOUNDS for MTTF and FR (Hint: Develop 80% CI)

Often, we just need a lower or upper bound on reliability. Assume we are interested in a 90% reliability lower bound for this case and we know mission time = 3000. We reestimate the failure rate bound, for the error α = 0.1, for only one side. With Degree of Freedom = 2n, the new Chi-square value:

$$\chi^2_{2n,1-\alpha} = \chi^2_{50,0.9} = 63.167$$

Hence, the corresponding for the mean life and failure rate are:

$$Mean = \frac{2T}{\chi^2_{2n,1-\alpha}} = \frac{142178}{63.167} = 2250.827$$

$$FR = \frac{1}{Mean\ Life} = \frac{1}{2250.827} = 0.0004443$$

Which, in turn, allow us to calculate 90% Lower Bound for the reliability "R" of the device.

$$R - lower(T) = P\{x \geq T\} = EXP\left\{-\frac{T}{\theta}\right\} = EXP\{-pT\}$$

$$= EXP\{-3000 \times 0.0004443\} = 0.264$$

2.2. With Truncated at the 7th Failure.

Data Display

The 7th ↓

Failure **(Sort smallest to largest)**

0.2	131.8	457.7	520.5	599.0	614.6	750.4	1132.1
1271.3	1602.9	1622.1	1758.9	1913.8	1946.9	2425.1	2478.7
2677.4	3207.3	4234.4	4442.3	4698.1	5120.5	5358.7	10460.5
11663.9							

2.2.1 95% Confidence Interval for the Mean Time to Failure (MTTF)

In this case, the experiment is truncated at the 7^{th} failure = 750.4. We have 7 failures with known failure time. The confidence interval is 95%. By adding all the known test times of failures and assumed failure time for the last 18 samples, we get total time is 16582.

$$k = 7$$

$$\alpha = 0.05$$

$$T = \sum_{i=1}^{7} T_i + (25 - 7) \times 750.4 = 16582$$

With these inputs, we can use the Chi-Squared distribution table to calculate the confidence interval in the experiment with the degree of freedom = 2k:

$$\chi^2_{2k,1-\frac{\alpha}{2}} = \chi^2_{14,0.975} = 26.119$$

$$\chi^2_{2k,\frac{\alpha}{2}} = \chi^2_{14,0.025} = 5.629$$

Hence, the corresponding for the mean life is:

$$\left(\frac{2T}{\chi^2_{2k,1-\frac{\alpha}{2}}}, \frac{2T}{\chi^2_{2k,\frac{\alpha}{2}}}\right) = \left(\frac{33164}{26.119}, \frac{33164}{5.629}\right) = (1269.741, 5891.970)$$

2.2.2 95% Confidence Interval for the Failure Rate (FR)

$$Failure\ Rate = \frac{1}{Mean\ Life}$$

$$= \left(\frac{1}{5891.970}, \frac{1}{1269.741}\right) = (0.0001697, 0.0007876)$$

2.2.3 90% Confidence BOUNDS for MTTF and FR (Hint: Develop 80% CI)

In a 90% reliability lower bound with mission time = 3000. For the error α = 0.1, for only one side, with Degree of Freedom = 2k, the new Chi-square value:

$$\chi^2_{2k,1-\alpha} = \chi^2_{14,0.9} = 21.064$$

Hence, the corresponding for the mean life and failure rate are:

$$Mean = \frac{2T}{\chi^2_{2k,1-\alpha}} = \frac{33164}{21.064} = 1574.443$$

$$FR = \frac{1}{Mean\ Life} = \frac{1}{1574.443} = 0.0006351$$

Which, in turn, allow us to calculate 90% Lower Bound for the reliability "R" of the device.

$$R - lower(T) = P\{x \geq T\} = EXP\left\{-\frac{T}{\theta}\right\} = EXP\{-pT\}$$

$$= EXP\{-750.4 \times 0.0006351\} = 0.621$$

2.3. With Truncated at time T= 0.25*MTTF.

Data Display

Time=750

```
Failure (Sort smallest to largest)
   0.2     131.8     457.7     520.5     599.0     614.6     750.4    1132.1
1271.3    1602.9    1622.1    1758.9    1913.8    1946.9    2425.1    2478.7
2677.4    3207.3    4234.4    4442.3    4698.1    5120.5    5358.7   10460.5
11663.9
```

2.3.1 95% Confidence Interval for the Mean Time to Failure (MTTF)

In this case, the experiment runs in a specific time = 0.25*3000 = 750, so we have 6 failures with known failure time. The confidence interval is 95%. We get total time is T = 18750.

$$k = 6$$

$$\alpha = 0.05$$

$$T = 750 \times 25 = 18750$$

With these inputs, we can use the Chi-Squared distribution table to calculate the confidence interval in the experiment with the degree of freedom = 2n+2:

$$\chi^2_{2k+2,1-\frac{\alpha}{2}} = \chi^2_{14,0.975} = 26.119$$

$$\chi^2_{2k+2,\frac{\alpha}{2}} = \chi^2_{14,0.025} = 5.629$$

Hence, the corresponding for the mean life is:

$$\left(\frac{2T}{\chi^2_{2n+2,1-\frac{\alpha}{2}}}, \frac{2T}{\chi^2_{2n+2,\frac{\alpha}{2}}}\right) = \left(\frac{37500}{26.119}, \frac{37500}{5.629}\right) = (1435.739, 6662.253)$$

2.3.2 95% Confidence Interval for the Failure Rate (FR)

$$Failure\ Rate = \frac{1}{Mean\ Life}$$

$$= \left(\frac{1}{6662.253}, \frac{1}{1435.739}\right) = (0.0001501, 0.0006965)$$

2.3.3 90% Confidence BOUNDS for MTTF and FR (Hint: Develop 80% CI)

In a 90% reliability lower bound with mission time = 18750. For the error α = 0.1, for only one side, with Degree of Freedom = 2k+2, the new Chi-square value:

$$\chi^2_{2k+2,1-\alpha} = \chi^2_{14,0.9} = 21.064$$

Hence, the corresponding for the mean life and failure rate are:

$$Mean = \frac{2T}{\chi^2_{2k+2,1-\alpha}} = \frac{37500}{21.064} = 1780.276$$

$$FR = \frac{1}{Mean\ Life} = \frac{1}{2021.634} = 0.0005617$$

Which, in turn, allow us to calculate 90% Lower Bound for the reliability "R" of the device.

$$R - lower(T) = P\{x \geq T\} = EXP\left\{-\frac{T}{\theta}\right\} = EXP\{-pT\}$$

$$= EXP\{-750 \times 0.0005617\} = 0.656$$

3. Summary

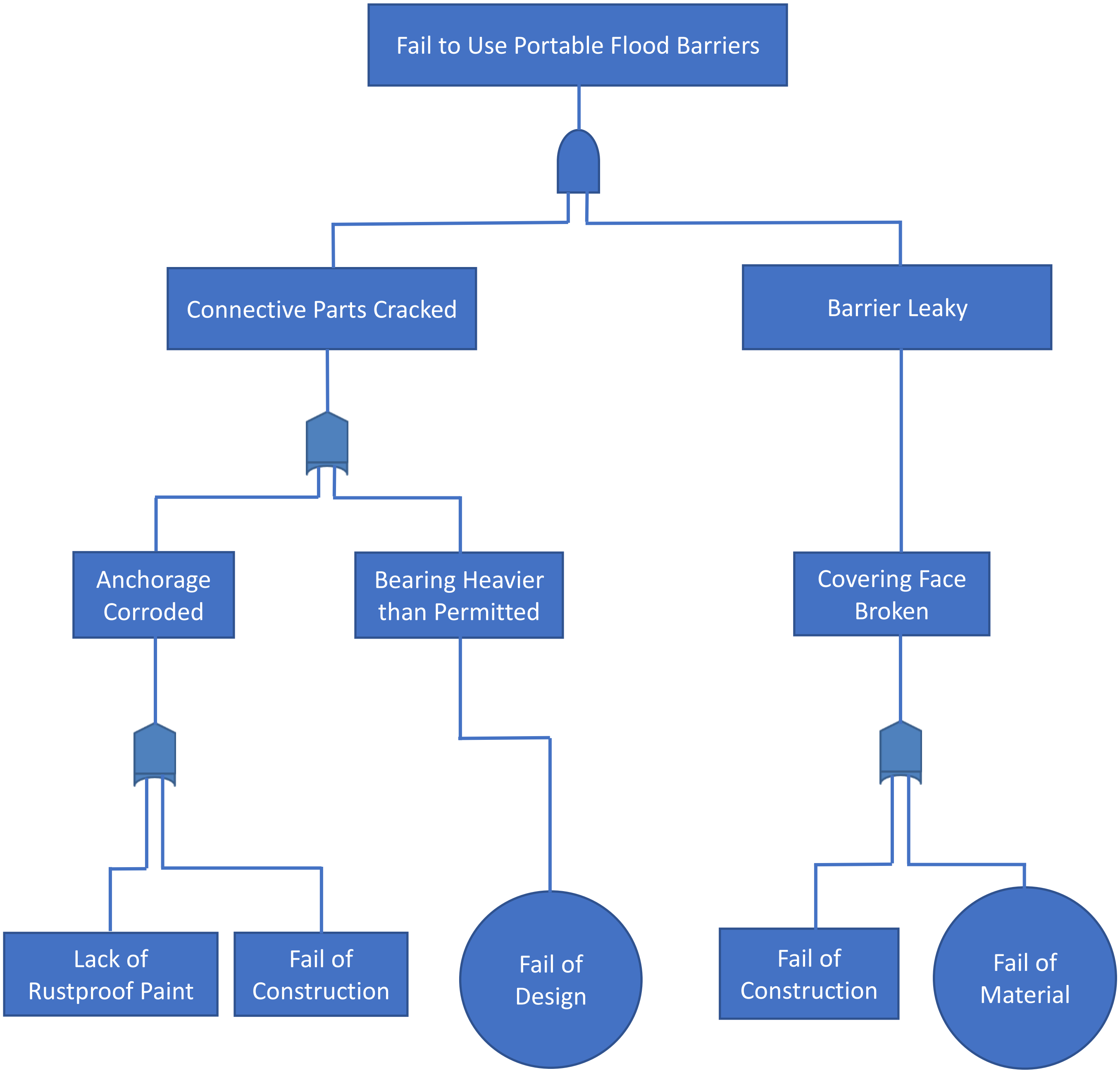
Collect the three sets of results in a table and compare.

	Complete	Truncated at 7^{th}	Truncated at 0.25 MTTF
95% CI for the MTTF	(1990.729, 4394)	(1269.741, 5891.97)	(1435.739,6662.253)
95% CI for the FR	(0.0002276, 0.0005023)	(0.0001697, 0.0007876)	(0.0001501,0.0006965)
90% Lower Confidence Bounds for the MTTF & FR	MTTF: 2250.827 FR: 0.0004443	MTTF: 1574.443 FR: 0.0006351	MTTF: 1780.276 FR: 0.0005617

As we can see, the complete data has the most accurate 95% confidence interval and failure rate. The failure and time censored (truncated) data, which are both incomplete, have larger confidence interval and larger failure rate.

4. Implement

4.1 A Fault Tree Analysis for Flood Barrier



4.2 A Failure Mode and Effects Analysis for Flood Barrier

Part / Design Parameter	Potential Failure Mode	Potential Failure Effects	SEV	Potential Causes	OCC	Current Controls	DET	RPN	Actions Recommended
Portable Flood Barrier	Connective part crack	Barrier destroy	10	Part Corroded	2	Protective paint	1	20	Redesign the structure of connective part
	Barriers leaky	Barrier can't be used	8	Damaged during delivery	5	Strength the package	10	400	Add protections during delivery
	Missing part	Barrier can't be used	6	Fail to check	6	Offer backup parts	5	180	Set up double-check mechanism
	Do not reach 1.5m height	Barrier can't prevent flood	5	Installed improperly	2	Offer detailed Instruction	3	30	Training the QA department
After-sale service	Service hotline is always busy	Customers disappointed	3	Hotlines are few	10	Set more hotlines	9	270	Invest more money on After-sale apartment

Severity (SEV)		
If SEV >=	8	
Otherwise		
If SEV <=	3	

Risk Priority Number (RPN)		
If RPN >=	300	
Otherwise		
If RPN <=	100	

Appendix A:

MFE634

Group Take Home Final Exam Q-2: S2018

Group No:________________________________ Date: _____________________

Reliability Analysis Question: use the following Group Exp. MTTF:

Group	Expon. MTTF	Group	Expon. MTTF
One	**2,500 hours**	All others	2.5K+GNo*100

Generate 25 exponentially distributed times to failure, with MTTF as defined above. Using as a Model the START sheets on Exponential Reliability and Censored Data
Analyze such above defined **complete sample**, in the following manner:

a) Obtain a 95% Confidence Interval for the Mean Time to Failure (MTTF)
b) Obtain a 95% Confidence Interval for the Failure Rate (FR)
c) Obtain 90% the Confidence BOUNDS for MTTF and FR (Hint: Develop 80% CI)

Now, assume your sample has been **"Truncated" at the 7th failure**. This means the test was suspended at that Failure Time. Thence, you will never know when the remaining units will fail. With such **Failure Censored Data**:

a) Obtain a 95% Confidence Interval for the Mean Time to Failure (MTTF)
b) Obtain a 95% Confidence Interval for the Failure Rate (FR)
c) Obtain 90% Confidence BOUNDS for MTTF and FR (Hint: Develop 80% CI)

Finally, assume your sample has been **"Truncated"** at time **T= 0.25*MTTF**. This means the test was suspended after that Truncation Time. Thence, you will never know when units still working after such time T, will fail. With such **Time Censored Data**:

a) Obtain a 95% Confidence Interval for the Mean Time to Failure (MTTF)
b) Obtain a 95% Confidence Interval for the Failure Rate (FR)
c) Obtain 90% Confidence BOUNDS for MTTF and FR (Hint: Develop 80% CI)

Compare the three sets of results in a table. Take Conclusions.

Now, implement (1) a Fault Tree Analysis (FTA) and (2) a Failure Mode and Effects Analysis (FMEA) to an example of your Group Project Topic problem. Make it specific to your Topic problem issues. Use the Blueprints material in BB as a guide.

This Take Home Part of the Final Exam is worth 10 Points (Companion, also 10 points).

Each Group will deliver a printout/file of this Take Home at Entrance of the Final Exam.