Using Group Explorer in Teaching Abstract Algebra

Claus Schubert, Mary Gfeller, Christopher Donohue

Abstract

This study explores the use of Group Explorer in an undergraduate mathematics course in abstract algebra. The visual nature of Group Explorer in representing concepts in group theory is an attractive incentive to use this software in the classroom. However, little is known about students’ perceptions on this technology in learning concepts in abstract algebra. Twenty-six participants in an undergraduate course studying group theory were surveyed regarding their experiences using Group Explorer. Findings indicate that all participants believed that the software was beneficial to their learning and described their attitudes regarding the software in terms of using the technology and its helpfulness in learning concepts. A multiple regression analysis reveals that representational fluency of concepts with the software correlated significantly with participants’ understanding of group concepts yet, participants’ attitudes about Group Explorer and technology in general were not significant factors.

Keywords

Visualization, group theory, Group Explorer, abstract algebra, perceptions, technology

This is an Author's Original Manuscript of an article whose final and definitive form, the Version of Record, has been published in the International Journal of Mathematical Education in Science and Technology 44(3)(2013) (copyright Taylor & Francis), available online at: http://www.tandfonline.com/doi/abs/10.1080/0020739X.2012.729680.
Introduction

One of the most notable features in the teaching and learning of undergraduate abstract algebra is students’ difficulty in grasping the abstract nature of the course and its concepts. Several researchers have suggested ways to develop these concepts (Dubinsky, Dautermann, Leron & Zazkis, 1994; Edwards & Brenton, 1999; Hazzan, 1999; Zazkis, Dubinsky, & Dautermann, 1996) with most of the attention residing with how students typically come to understand concepts such as groups, subgroups, and quotient groups. Within this body of research, technology has also played a role in the examination of the teaching and learning undergraduate abstract algebra (Leron & Dubinsky, 1995). The purpose of this study is to investigate the use of Group Explorer in an undergraduate abstract algebra classroom and its role in expanding students’ representations of group theory concepts.

Theoretical Framework

Developing and understanding concepts in undergraduate mathematics courses is a difficult task for many students. As Tall (1992) pointed out, conceptual understanding in advanced mathematics often requires students to examine and make sense of the cognitive structures which form the relationships among various concepts through concept image and concept definition. Past research on the learning of undergraduate abstract algebra focuses mainly on concept development but also includes a framework of action, process, object, and schema (Dubinsky et al., 1994). In this framework, an action is a repeatable transformation of an object, physical or mental. When the action has been mastered to a level where the individual can complete this in their mind, it has become a process. The process itself can then be manipulated to obtain new processes, which makes it an object. A schema is a collection of processes and objects. Hazzan (1999) describes how students reduce the level of abstraction to help their learning. Strategies include relying on specific examples of abstract structures, such as integers as an example of a group, and changing language such as “there exists” to “I can find” to make the student feel more involved with the problem.

Edwards and Brenton (1999) describe an alternative method of teaching abstract algebra than the
usual theorem-proof-corollary approach using concrete examples to motivate a new concept, such as a penny moving on a 2x2 checkerboard to introduce the Klein Vier group. These were implemented in a class and students filled out a questionnaire at the end of the class which described these methods as helpful. However, the authors warn that over-simplification of mathematical concepts may deprive students of the needed practice to turn processes into objects. Zazkis et al. (1996) discuss connections between visual and analytic approaches to abstract algebra using the specific example of the dihedral group D₄.

Maybe not surprisingly to anyone teaching abstract algebra, one recurring theme is the learner’s difficulty in grasping the concept of quotient group. There is a further level of abstraction involved as a quotient group is a set of sets. Various authors also mention conceptual problems students have with cyclic groups (Dubinsky et al., 1994, Edwards & Brenton, 1999, Hazzan, 1999). For example, the question "Is Z₃ a subgroup of Z₆?" poses some difficulty. These two problems can be viewed as related. As Edwards and Brenton (1999) point out, viewing the elements of Zₙ as integers rather than equivalence classes of integers is an over-simplification that leads to confusion about whether Z₃ is even a subset of Z₆. Viewing Zₙ instead as a quotient group Z/nZ from the start may help alleviate this particular problem.

Leron and Dubinsky (1995) describe using the programming language ISETL (Interactive SET Language) in lieu of traditional lectures to teach abstract algebra concepts and how it benefits student learning. Students worked on computer exercises in teams and were guided to concepts and results this way. The authors claim that this environment is more conducive to the development of mental constructions corresponding to processes and objects and believe it should entirely replace the traditional lecture, although short discussions and summaries as well as traditional homework assignments are still used.

Understanding mathematical concepts also requires the use of various representations that are intended to evoke meaning for both mathematical concepts and processes. Useful representations in mathematics include the symbolic meanings through algebraic manipulation as well as graphs and diagrams. According to NCTM (2000), representation plays an essential and fundamental role learning mathematics and is defined as “the act of capturing a mathematical concept or relationship in some form” (p. 67). Research on the use of technology in mathematics education
has generally focused on understanding the potential benefits of technology in the classroom as well as pitfalls. One of the most beneficial aspects of technology for learning mathematics is the understanding of mathematical concepts through a variety of representations, including numerical, symbolic, and graphical (visual) representations (Kaput, 1989). Dick (2007) uses the term “representational fluency” to refer to the learner’s ability to understand mathematical concepts through various representations. For example, there are many ways to conceptualize the absolute value of a number: 1) the distance between the number and zero, 2) the square root of the square of the number, and 3) a piece-wise defined function whereby the absolute value of the number is the number itself when the number is positive or the negative of the number when the number is negative.

Current efforts in teaching abstract algebra with technology have included the use of visual representations through the program Group Explorer. However, little research has been conducted on the visual representation of basic group theory concepts. Thus, the purpose of this study is to investigate the use of Group Explorer in an undergraduate abstract algebra class. The research questions are:

1. How do students perceive the use of Group Explorer in learning abstract algebra?
2. What are students’ understandings of cyclic groups, normal subgroups, and quotient groups from the visual representation of these concepts in Group Explorer?
3. What relationships, if any, exist among students’ perceived benefits of the use of Group Explorer, representational fluency and conceptual understanding?

Methods

**Group Explorer**

The software investigated in this study is called Group Explorer. Group Explorer is a freely available software that helps students visualize groups and subgroups with operation tables, Cayley diagrams, cycle graphs, and symmetry objects. A group is a basic algebraic structure usually introduced to undergraduate mathematics students in an upper division mathematics class. Any set with an operation that satisfies the associative law, has a neutral element, and an inverse element for every given element is a group. Every group of order up to 20 has its own
information sheet in Group Explorer with all pertinent data such as being Abelian (the operation is commutative), cyclic (the group can be generated by powers of a single element), orders (cardinalities) of all subgroups, as well as some advanced topics such as being solvable or the class equation (see Figure 1).

Figure 1. Information sheet for the group $S_3$ in Group Explorer.
This software was used by the first author as the instructor for an undergraduate abstract algebra course. Students were asked to use the software as part of their homework assignment to answer various questions related to current class topics. Detailed worksheets for students to follow were provided. These worksheets were designed by the first author and included some basic questions about specific groups, as well as some guided discovery questions. For example, students were asked to make a conjecture about the relationship between the order of a group and the orders of its subgroups after analyzing the number of elements in several cases. Another assigned problem was to determine several quotient groups by sorting a specific multiplication table by a specific coset. Figures 2 and 3 show how students can visualize that the quotient group of the dihedral group $D_8$ of order 16 by the cyclic subgroup generated by $r^2$ of order 4 is isomorphic to the Klein group with four elements. The dihedral group $D_8$ is the group of symmetries of a regular octagon and is generated by two elements, a rotation by 45 degrees ($r$) and a flip ($f$). The direction of the rotation and the axis for the flip do not matter in terms of the group structure. The fact that the group has 16 elements and the subgroup generated by $r^2$ has 4 elements easily establishes that the quotient group has $16/4=4$ elements. However, there are two nonisomorphic groups of order 4 and without further computation it is not obvious which of the two is the correct one. By looking at the patterns of the multiplication tables of the original group sorted by the given subgroup, and the possible groups of order 4, students can easily establish the correct answer.

**Participants**

The participants were 26 mathematics and mathematics education majors enrolled in two sections of an abstract algebra course at a small four-year college located in the North eastern United States. The course, taught by the first author, is required for both mathematics and mathematics education majors. Participants were recruited at the beginning of the course after they were introduced to Group Explorer. All of the participants who volunteered remained in the study until its completion at the end of the semester.
Figure 2. Group Explorer view of the multiplication table of the dihedral group $D_{10}$, separated by cosets of the subgroup generated by $r^2$.

<table>
<thead>
<tr>
<th></th>
<th>$e$</th>
<th>$r$</th>
<th>$r^2$</th>
<th>$r^3$</th>
<th>$r^4$</th>
<th>$r^5$</th>
<th>$r^6$</th>
<th>$r^7$</th>
<th>$r^8$</th>
<th>$r^9$</th>
<th>$r^{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>$e$</td>
<td>$r$</td>
<td>$r^2$</td>
<td>$r^3$</td>
<td>$r^4$</td>
<td>$r^5$</td>
<td>$r^6$</td>
<td>$r^7$</td>
<td>$r^8$</td>
<td>$r^9$</td>
<td>$r^{10}$</td>
</tr>
<tr>
<td>$r$</td>
<td>$r$</td>
<td>$r^2$</td>
<td>$r^4$</td>
<td>$r^6$</td>
<td>$r^8$</td>
<td>$r^{10}$</td>
<td>$e$</td>
<td>$r^2$</td>
<td>$r^4$</td>
<td>$r^6$</td>
<td>$r^8$</td>
</tr>
<tr>
<td>$r^2$</td>
<td>$r^2$</td>
<td>$r^4$</td>
<td>$r^6$</td>
<td>$r^8$</td>
<td>$r^{10}$</td>
<td>$e$</td>
<td>$r^2$</td>
<td>$r^4$</td>
<td>$r^6$</td>
<td>$r^8$</td>
<td>$r^{10}$</td>
</tr>
<tr>
<td>$r^3$</td>
<td>$r^3$</td>
<td>$r^6$</td>
<td>$r^9$</td>
<td>$r^2$</td>
<td>$r^5$</td>
<td>$r^8$</td>
<td>$r^1$</td>
<td>$r^4$</td>
<td>$r^7$</td>
<td>$r^{10}$</td>
<td>$e$</td>
</tr>
<tr>
<td>$r^4$</td>
<td>$r^4$</td>
<td>$r^8$</td>
<td>$r^2$</td>
<td>$r^6$</td>
<td>$r^{10}$</td>
<td>$e$</td>
<td>$r^2$</td>
<td>$r^4$</td>
<td>$r^6$</td>
<td>$r^8$</td>
<td>$r^{10}$</td>
</tr>
<tr>
<td>$r^5$</td>
<td>$r^5$</td>
<td>$r^{10}$</td>
<td>$e$</td>
<td>$r^2$</td>
<td>$r^5$</td>
<td>$r^8$</td>
<td>$r^{1}$</td>
<td>$r^4$</td>
<td>$r^7$</td>
<td>$r^{10}$</td>
<td>$e$</td>
</tr>
<tr>
<td>$r^6$</td>
<td>$r^6$</td>
<td>$e$</td>
<td>$r^2$</td>
<td>$r^4$</td>
<td>$r^6$</td>
<td>$r^8$</td>
<td>$r^{10}$</td>
<td>$e$</td>
<td>$r^2$</td>
<td>$r^4$</td>
<td>$r^6$</td>
</tr>
<tr>
<td>$r^7$</td>
<td>$r^7$</td>
<td>$r^4$</td>
<td>$r^6$</td>
<td>$r^8$</td>
<td>$r^{10}$</td>
<td>$e$</td>
<td>$r^2$</td>
<td>$r^4$</td>
<td>$r^6$</td>
<td>$r^8$</td>
<td>$r^{10}$</td>
</tr>
<tr>
<td>$r^8$</td>
<td>$r^8$</td>
<td>$r^{10}$</td>
<td>$e$</td>
<td>$r^2$</td>
<td>$r^5$</td>
<td>$r^8$</td>
<td>$r^{1}$</td>
<td>$r^4$</td>
<td>$r^7$</td>
<td>$r^{10}$</td>
<td>$e$</td>
</tr>
<tr>
<td>$r^9$</td>
<td>$r^9$</td>
<td>$r^2$</td>
<td>$r^4$</td>
<td>$r^6$</td>
<td>$r^8$</td>
<td>$r^{10}$</td>
<td>$e$</td>
<td>$r^2$</td>
<td>$r^4$</td>
<td>$r^6$</td>
<td>$r^8$</td>
</tr>
<tr>
<td>$r^{10}$</td>
<td>$r^{10}$</td>
<td>$r^{10}$</td>
<td>$e$</td>
<td>$r^2$</td>
<td>$r^5$</td>
<td>$r^8$</td>
<td>$r^{1}$</td>
<td>$r^4$</td>
<td>$r^7$</td>
<td>$r^{10}$</td>
<td>$e$</td>
</tr>
</tbody>
</table>

Figure 3. Group Explorer view of the multiplication table of the Klein group $V_4$, which is isomorphic to the quotient group depicted in Figure 2.
Design of the Study

This study utilizes both qualitative and quantitative design. Participants in the course were required to use the software Group Explorer to investigate concepts of group theory. Participants were given worksheets to complete outside class in the computer lab or at home. These exercises covered concepts such as Lagrange's theorem, normal subgroups and quotient groups. At the end of the semester, participants were surveyed through a questionnaire about their experiences with the software and technology in general, and their understanding of how to use the technology to answer specific questions about a few of the concepts mentioned above. Thus, the questionnaire was developed specifically for this study and contains three sections.

In the first section, two questions measure participants’ representational fluency with regards to basic group theory concepts including cyclic and Abelian groups, and advanced topics including normal subgroups, factor groups, and isomorphism. The second section on the questionnaire provides a likert scale designed to measure participants’ attitudes toward Group Explorer as a beneficial technological tool for learning and their attitudes toward technology in general. The questionnaire was adapted and modified from Kay and Knaack (2007). The wording was changed to reflect the particular technology used in all but one item from the original instrument. One item was removed from the original instrument (Item 4) because it is unlikely that participants would use this technology in any other subsequent mathematics course. In the third section, the final two questions ask participants to discuss their experience with Group Explorer and any perceived benefits. Participants completed the questionnaire within 30 minutes. Participants’ responses to questions on their final exam relating to the concept explored for basic and advanced representational fluency were recorded and coded for correctness.

Data Analysis

The analysis began with coding of the content questions. The first author coded the two questions in the first section of the questionnaire pertaining to representational fluency in the following way: Cyclic/Abelian (4 = Conceptual and visual understanding, 3 = Conceptual understanding but no visual, or, minor flaws, 2 = Conceptual understanding but with flaws, no understanding of the visualization, major flaws, 1 = No understanding of the concept) and
Normal/Isomorphic (4 = Full conceptual understanding and knew how to check visually, 3 = Full conceptual understanding but did not know how to check OR one of two parts perfect, 2 = Partial conceptual understanding and/or did not know how to check visually, 1 = No understanding of concepts).

For the first content question, participants were asked the following: Describe how you could tell in Group Explorer if a given group is (a) cyclic, (b) Abelian.

The following provides sample scored response of each score:

(a) If the group diagram is a complete circle. (b) If the group table is symmetric about the line made from the top left corner to the bottom right corner. (Participant 7, 4 points)

(a) You could tell that the group was cyclic by looking at the subgroups and seeing if one subgroup generated the whole subgroup. (b) If, when looking at the multiplication table, the table was symmetric along the diagonal then you knew the group was Abelian. (Participant 24, 3 points)

(a) In group explorer you can tell that a group is cyclic if the colors of the boxes are symmetric (commutative). (b) I’m pretty sure I would tell that the same way I could tell if it was cyclic. (Participant 18, 1 point)

For the second content question, participants were asked the following: Suppose G is a group and H is a normal subgroup of G. (a) How could you use Group Explorer to visualize that H is normal in G? (b) How would you determine which group G/H is isomorphic to in Group Explorer?

The following provides sample scored responses of each score:

(a) By using the multiplication table and sorting the table by the subgroup H and separating the cosets. Then I would look at the colored blocks for symmetry, as well as, color patterns to visualize that H is normal. (b) By following the same procedure mentioned in (a), but instead I would use each of the colored blocks as one element in the new set. Then I would count the number of elements and compare the multiplication table with the multiplication table of another group of the same order to determine if they are similar in pattern. (Participant 21, 4 points)
(a) We used the Cayley table & rearranged it by subgroups. Then we split the table (by moving the arrow to the right). If it split into 4 boxes then the subgroup is normal. (b) I would look at the Cayley table of G/H and compare it to the Cayley table of other groups. (Participant 9, 2 points)

(a) By looking at the size of both and components in both groups. (b) There is a drop list that shows you which groups are isomorphic to each other. Also by looking at the Cayley tables and diagrams. (Participant 17, 1 point)

The data was re-coded several months later using a slightly different coding scheme in order to achieve reliability of a single rater. In this scheme, a score for each question was given based on algebraic understanding and visual understanding of the concepts (4 being the highest and 1 being the lowest). An average of these two scores was taken and compared to the original scoring through a paired t-test. The results indicated no significant difference between the results (p=.26, α=0.01), indicating a high level of reliability was achieved.

The second section of the questionnaire was analyzed using percentages. Items were rated on a scale of 1(unfavorable) to 7 (favorable), with the exception of Item 3, which used reverse coding. Items 1, 2, and 3 (with reverse coding) were added together to get a measure of participants’ attitudes toward Group Explorer. Items 4 and 5 were added together to serve as a measure of attitudes toward technology in general. Item 6, which was intended to reflect views about technology, was removed from all analyses due to lack of clarity.

The third section of the questionnaire was coded by two authors independently using analytic induction. After coding, each author established categories for the responses using constant comparison methods. Approximately 80% of the codes given by the two researchers were in agreement. After discussion, 100% agreement was reached regarding the categories that were formed. Finally, items from participants’ scores on exam questions regarding the concepts of cyclic, Abelian, normal, and isomorphic were analyzed by the first author and recorded.
Results

In the written responses, all 26 participants viewed Group Explorer as beneficial. Table 1 shows the percentages of favorable scores for the items on the survey.

Table 1

Results of Group Explorer Survey

<table>
<thead>
<tr>
<th>Question</th>
<th>% marking</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Group Explorer has some benefit in terms of providing me with another learning strategy.</td>
<td>88</td>
</tr>
<tr>
<td>2. I feel Group Explorer did benefit my understanding of the concepts we learned in class.</td>
<td>73</td>
</tr>
<tr>
<td>3. I did not benefit from using Group Explorer.</td>
<td>81*</td>
</tr>
<tr>
<td>4. I enjoy working on the computer to learn.</td>
<td>50</td>
</tr>
<tr>
<td>5. I find computer graphics and interactive programs help me learn.</td>
<td>50</td>
</tr>
<tr>
<td>6. I benefit from learning through interactive and engaging activities.</td>
<td>85</td>
</tr>
</tbody>
</table>

In addition to the counts above, participants’ descriptions of their likes and dislikes about Group Explorer were collected. Two categories of responses that emerged in describing what participants liked about Group Explorer were Technology and Learning. In the Technology category, participants referred to 1) the ease of using the technology, 2) the easy access to group characteristics, and 3) its dynamic capabilities.

I liked how easy GE was to learn to use, made it easier to work with. It was useful that anything you wanted to know about groups whether it was a definition of a term, how to determine something (ex: isomorphism) there was a library for that. I didn’t have my book next to me to refer to everything. Everything I needed was on GE. (Participant 1)
I liked seeing visually what we were trying to determine in class. It made it easier to see why certain rules are followed and why certain concepts pertain to certain problems. (Participant 2)

I liked the ease of manipulating the tables and Cayley diagrams. Being able to rename the elements of the groups and change generators was helpful. Generating and highlighting cosets was also very helpful. (Participant 4)

I liked that it’s always available and you can change things to suit your needs. (Participant 8)

I liked that it was outside the norm of the usual classroom. Class can get kind of boring and routine so having something different is nice. (Participant 22)

In the Learning category, participants referred to 1) the enhancement of visual learning 2) the support for concept formation, 3) the reinforcement of prior concepts, and 4) its contribution to student motivation:

I liked that I could visually see the groups with the multiplication tables. It made it easier to understand what was going on with groups. I also liked the page that showed all of the facts about the groups. If I wasn’t sure of something the answer was visually on there. I also liked the assignments. It was a nice change to book work and I looked forward to doing them. (Participant 19)

Overall, I had great difficulty in this class. I liked Group Explorer due to the fact that it allowed me more opportunity to review some of the basic concepts trying to be explained outside of class. (Participant 21)

Made me more familiar with a lot of the groups and how to identify them. (Participant 25)

Two major themes emerged when examining the participants’ dislikes about Group Explorer. First, a majority of participants had trouble with the labeling of elements in Group Explorer. Even though participants were shown how to change the labels, participants were bothered by the different variables used for the same element. Second, some participants felt overwhelmed with all of the tools available to them. In addition to the two major themes stated here, two
participants alluded to the inability to use Group Explorer as a tutorial because the software was not capable of generating problems or allowing users to input information to be evaluated.

A multiple regression analysis was used to explore whether a relationship exists among the variables of representational fluency (combining the basic and advanced fluency scores), attitudes toward Group Explorer and attitudes toward technology in general and participants’ concept understanding, which was determined by participants’ final exam score. Since a small sample was acquired for this study, a .01 level of significance was used for rejecting the null hypothesis to reduce Type I errors. Table 2 shows the correlation matrix of all variables, revealing a moderate correlation between total representational fluency and concept understanding ($r = .42$).

**Table 2**  
*Matrix correlation between technology variables and concept understanding*

<table>
<thead>
<tr>
<th></th>
<th>Group Explorer</th>
<th>Technology</th>
<th>Concept Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Representational Fluency</td>
<td>.35</td>
<td>.14</td>
<td>.49*</td>
</tr>
<tr>
<td>Group Explorer</td>
<td></td>
<td>.38</td>
<td>.27</td>
</tr>
<tr>
<td>Technology</td>
<td></td>
<td></td>
<td>-.23</td>
</tr>
</tbody>
</table>

Table 3 displays three regression models with beta estimates, standard errors, and standardized beta estimates. Representational fluency correlated significantly with participants’ concept understanding. Attitudes about Group Explorer and technology were not significantly correlated to concept understanding; however, the third model indicates a moderate association between the independent and dependent variables.
Table 3

*Three Multiple Regression Models with Concept Understanding as Dependent Variable (N=26)*

<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th>SE B</th>
<th>B</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Representational Fluency</td>
<td>5.96</td>
<td>2.12</td>
<td>.49*</td>
<td>.25</td>
</tr>
<tr>
<td><strong>Model 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Representational Fluency</td>
<td>6.49</td>
<td>2.05</td>
<td>.54</td>
<td>.34</td>
</tr>
<tr>
<td>Technology</td>
<td>-2.77</td>
<td>1.55</td>
<td>-.31</td>
<td></td>
</tr>
<tr>
<td><strong>Model 3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Representational Fluency</td>
<td>5.57</td>
<td>2.13</td>
<td>.46</td>
<td>.39</td>
</tr>
<tr>
<td>Technology</td>
<td>-3.54</td>
<td>1.63</td>
<td>-39</td>
<td></td>
</tr>
<tr>
<td>Group Explorer</td>
<td>4.42</td>
<td>3.35</td>
<td>.25</td>
<td></td>
</tr>
</tbody>
</table>

**Discussion and Conclusion**

The results of this study indicate that participants’ representational fluency was a significant factor in determining concept understanding at the end of the course. It should be noted that due to the design of this study, the results are not generalizable. Nevertheless, the findings suggest that this group of undergraduate students were highly receptive to using Group Explorer, found the software easy to use, and believed it to be a useful tool for visualizing concepts in abstract algebra. Many of the participants in this study identified with visual learning as a positive aspect of Group Explorer, yet there seemed to be little evidence that liking Group Explorer actually aids in concept understanding.

The hope for technological tools in the mathematics classroom is to provide a more meaningful experience to the learner and to broaden the learner’s representational fluency. NCTM (2000) emphasizes the importance of visual representations heavily, and the authors believe this
recommendation should be extended to college level mathematics. Representation is an important tool that helps students to organize their thinking, and “good representations can help teachers represent abstract concepts in a more concrete way” (NCTM, 2000, p. 68). One of the most striking attributes of Group Explorer is its ability to produce highly colorful, visual displays for abstract concepts. Visual and graphical representations have been a central component of mathematics education for centuries, yet the use of color has only been introduced recently as a method to distinguish mathematical concepts. Gardner (1993) defines a visual learner as one who relates meanings to spatial relations and color. Currently, the phrase visual literacy has emerged as a 21st century skill across all academic disciplines, including mathematics. In a world where visual representations have become exponentially more prevalent, visual literacy is more fundamental to a person’s ability to effectively interact in society than ever before. Felton (2008) describes visual literacy as a person’s ability to “understand, produce, and use culturally significant images, objects, and visible actions” (p. 60). For the purposes of our study, Felton’s description should be viewed through the lens of a mathematics classroom culture, comprised of mathematicians, math educators and their students. Over the centuries, images emerged from the hybridization of the exacting definitions and representations employed by professional mathematicians, and the efforts of educators to simplify and streamline this mathematical language to make it more accessible to their students.

In continuing to investigate this particular technology, future research on the use of Group Explorer in undergraduate abstract algebra courses should include a more in-depth look at how the learner experiences the software in learning various representations of abstract algebra concepts. As Arcavi (2003) suggested, the cultural, cognitive, and socio-cultural aspects of visualization should be considered when examining technology and should be considered in further explorations of Group Explorer. Thus, future examinations of Group Explorer could investigate how learners apply the various representations to problems and to problem solving activities, as well as the role of the instructor in providing a flexible learning environment.
References


