

Thermodynamics and Isotopes in Geology, Problem 7

**A preliminary phase diagram for the Al<sub>2</sub>SiO<sub>5</sub> system**

Kyanite, andalusite, and sillimanite are very important metamorphic minerals found in phyllites, schists, and gneisses. You can use thermodynamic data and equations to determine the location (in PT space) of phase boundaries for these minerals. For this exercise, we are assuming that  $\Delta V^{\circ}_{\text{rxn}}$ ,  $\Delta H^{\circ}_{\text{rxn}}$ , and  $\Delta S^{\circ}_{\text{rxn}}$  remain constant at high temperature and pressure.

**Step 1:**

form	formula	$\Delta H^{\circ}$ KJ/mol	$\Delta G^{\circ}$ KJ/mol	$S^{\circ}$ J/mol/K	$V^{\circ}$ cm <sup>3</sup> /mol
kyanite	Al <sub>2</sub> SiO <sub>5</sub>	-2594.29	-2443.88	83.81	44.09
andalusite	Al <sub>2</sub> SiO <sub>5</sub>	-2590.27	-2442.66	93.22	51.53
sillimanite	Al <sub>2</sub> SiO <sub>5</sub>	-2587.76	-2440.99	96.11	49.90

Fill in the chart below with changes in H<sup>o</sup>, G<sup>o</sup>, S<sup>o</sup>, and V<sup>o</sup> for each reaction. Note, the units for  $\Delta H^{\circ}_{\text{rxn}}$  and  $\Delta G^{\circ}_{\text{rxn}}$  should be convert from KJ/mol to J/mol and  $\Delta V$  should be converted from cm<sup>3</sup> to J/bar.

	$\Delta H^{\circ}_{\text{rxn}}$ (J/mol)	$\Delta G^{\circ}_{\text{rxn}}$ (J/mol)	$\Delta S^{\circ}_{\text{rxn}}$ (J/mol/K)	$\Delta V^{\circ}_{\text{rxn}}$ (J/bar)
1) kyanite → andalusite	_____	_____	_____	_____
2) andalusite → sillimanite	_____	_____	_____	_____
3) kyanite → sillimanite	_____	_____	_____	_____

**Step 2:**

Let's start with reaction (1); kyanite  $\rightarrow$  andalusite.

First, calculate  $\log K_{298}$  using the  $\Delta G^\circ_{\text{rxn}}$  calculated above.

Next, use the equation discussed in class to calculate the equilibrium temperature. List your final answer in both K and  $^\circ\text{C}$ .

Use the Clapeyron Equation to calculate the slope of this reaction (bars/K).

If you add 200 K (or  $^\circ\text{C}$ ) to the equilibrium temperature, what is the equilibrium pressure?

Plot these two points on the graph provided. Don't draw a line through them just yet....

**Step 3:**

Let's now examine reaction (2); andalusite  $\rightarrow$  sillimanite.

First, calculate  $\log K_{298}$  using the  $\Delta G^\circ_{\text{rxn}}$  calculated above.

Next, use the equation discussed in class to calculate the equilibrium temperature. List your final answer in both K and  $^\circ\text{C}$ .

Use the Clapeyron Equation to calculate the slope of this reaction (bars/K).

If you subtract 100 K (or  $^\circ\text{C}$ ) from the equilibrium temperature, what is the equilibrium pressure?

Plot these two points on the graph provided. You'll draw a line through them in a minute....

Note, this reaction (reaction 2) and the one before it (reaction 1), define the stable limits of andalusite. Now, starting at low pressure, draw lines through both sets of data and carefully extrapolate both lines (upward) until they just intersect.

**Step 4:**

Lastly, let's examine reaction (3); kyanite  $\rightarrow$  sillimanite.  
First, calculate  $\log K_{298}$  using the  $\Delta G^\circ_{\text{rxn}}$  calculated above.

Next, use the equation discussed in class to calculate the equilibrium temperature. List your final answer in both K and  $^\circ\text{C}$ .

Note that this temperature falls in the andalusite stability field. This is because kyanite and sillimanite would be *metastable* under these conditions. Kyanite and sillimanite are stable together at higher pressure (above the andalusite stability field). So we will calculate the slope of the reaction boundary and we will project this at pressures above the andalusite stability field.

Use the Clapeyron Equation to calculate the slope of this reaction (bars/K).

If you add 200 K (or  $^\circ\text{C}$ ) to the equilibrium temperature, what is the equilibrium pressure?

If you add 400 K (or  $^\circ\text{C}$ ) to the equilibrium temperature, what is the equilibrium pressure?

Plot these two points on the graph provided. Draw a line through them and extrapolate (downward) toward the intersection of the two previous reaction boundaries. Label the stability fields for kyanite, andalusite, and sillimanite.

Do your three reaction boundaries meet at a single point? \_\_\_\_\_

If not, are they close? Approximately how many °C and bars off?

If so, what is the temperature and pressure of the triple point? T = \_\_\_\_\_ P = \_\_\_\_\_