Understanding and Using Availability

Jorge Luis Romeu, Ph.D.
ASQ CQE/CRE, & Senior Member
C. Stat Fellow, Royal Statistical Society
Past Director, Region II (NY & PA)
Director: Juarez Lincoln Marti Int’l Ed. Project
Email: romeu@cortland.edu
Web: http://www.linkedin.com/pub/jorge-luis-romeu/26/566/104/
Webinar: June 8, 2017
Webinar Take-Aways

• Understanding Availability from a practical standpoint
• Calculating different Availability ratings
• Practical and Economic ways of enhancing Availability
Summary

Availability is a performance measure concerned with assessing a maintained system or device, with respect to its ability to be used when needed. We overview how it is measured under its three different definitions, and via several methods (theoretical/practical), using both statistical and Markov approaches. We overview the cases where redundancy is used and where degradation is allowed. Finally, we discuss ways of improving Availability and provide numerical examples.
When to use Availability

• When system/device can fail and be repaired
  – During “maintenance”, system is “down”
  – After “maintenance”, system is again “up”

• Formal Definition: “a measure of the degree to which an item is in an operable state at any time.” (Reliability Toolkit, RIAC)
System Availability

• A probabilistic concept based on:
• Two Random Variables X and Y
  – X, System or device time between failures
  – Y, Maintenance or repair time
• Long run averages of X and Y are:
  – E(X) Mean time Between Failures (MTBF)
  – E(Y) Expected Maintenance Time (MTTR)
Availability by Mission Type

- **Blanchard** (Ref. 2): availability may be expressed differently, depending on the system and its mission. There are three types of Availability:
  - Inherent
  - Achieved
  - Operational
Inherent Availability: $A_i$

* Probability that a system, when used under stated conditions, will operate satisfactorily at any point in time.

* $A_i$ excludes preventive maintenance, logistics and administrative delays, etc.

$$A_i = \frac{MTBF}{MTBF + MTTR}$$
Achieved Availability: $A_a$

* Probability that a system, when used under stated conditions, will operate satisfactorily at any point in time, when called upon.
* $A_a$ includes other activities such as preventive maintenance, logistics, etc.
Operational Availability: $A_o$

* Probability that a system, when used under stated conditions will operate satisfactorily when called upon.

* $A_o$ includes **all factors** that contribute to system downtime (now called Mean Down Time, MDT) for **all** reasons (maintenance actions and delays, access, diagnostics, active repair, supply delays, etc.).
A_0 \text{ Long Run average formula:}

\[ A_0 = P\text{(System.Up)} = \frac{\text{Fav.Cases}}{\text{Tot.Cases}} \]
\[ = \frac{\text{Up.Time}}{\text{Cycle.Time}} = \frac{E(X)}{E(X) + E(Y)} \]
\[ = \frac{\text{MTBF}}{\text{MTBF} + \text{MDT}} \]
# Numerical Example

<table>
<thead>
<tr>
<th>Event</th>
<th>SubEvent</th>
<th>Time</th>
<th>Inherent</th>
<th>Achieved</th>
<th>Operational</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up</td>
<td>Running</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Down</td>
<td>Wait-D</td>
<td>10</td>
<td></td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Down</td>
<td>Diagnose</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Down</td>
<td>Wait-S</td>
<td>3</td>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Down</td>
<td>Wait-Adm</td>
<td>2</td>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Down</td>
<td>Install</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Down</td>
<td>Wait-Adm</td>
<td>3</td>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Up</td>
<td>Running</td>
<td>45</td>
<td>45</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>Down</td>
<td>Preventive</td>
<td>7</td>
<td></td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Up</td>
<td>Running</td>
<td>52</td>
<td>52</td>
<td>52</td>
<td>52</td>
</tr>
</tbody>
</table>

**UpTime**

<table>
<thead>
<tr>
<th></th>
<th>Inherent</th>
<th>Achieved</th>
<th>Operational</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>147</td>
<td>147</td>
<td>147</td>
</tr>
</tbody>
</table>

**Maintenance**

<table>
<thead>
<tr>
<th></th>
<th>Inherent</th>
<th>Achieved</th>
<th>Operational</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>13</td>
<td>20</td>
<td>38</td>
</tr>
</tbody>
</table>

**Availability**

<table>
<thead>
<tr>
<th></th>
<th>Inherent</th>
<th>Achieved</th>
<th>Operational</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.9188</td>
<td>0.8802</td>
<td>0.7946</td>
</tr>
</tbody>
</table>
Formal Definition of Availability

Hoyland et al (Ref. 1): availability at time \( t \), denoted \( A(t) \), is the probability that the system is functioning (up and running) at time \( t \).

\( X(t) \): the state of a system at time “t”
* “up” and running, \( X(t) = 1 \),
* “down” and failed \( X(t) = 0 \)

\( A(t) \) can then be written:
\[ A(t) = P\{X(t) = 1\}; \ t > 0 \]
Availability as a R. V.

\[ A = \frac{X}{X + Y}; X, Y > 0 \]

- The problem of obtaining the “density function” of A resolved via variable transformation of the joint distribution
- Based on the two Random Variables X and Y
time to failure X, and time to repair Y
- Expected and Variance of the Availability r.v.
- \( L_{10} \) (10th Percentile of A) = \( P\{A < 0.1\} = 0.1 \)
  - First and Third Quartiles of Availability, etc.
- Theoretical results, approximated by Monte Carlo
Monte Carlo Simulation

- Generate $n = 5000$ random Exponential failure and repair times: $X_i$ and $Y_i$
- Obtain the corresponding Availabilities: $A_i = \frac{X_i}{X_i + Y_i}; 1 \leq i \leq 5000$
- Sort them, and calculate all the $n = 5000$ $A_i$ results, numerically
- Obtain the desired parameters from them.
Numerical Example

• Use Beta distribution for expediency
  – Ratio yielding $A_i$ is distributed $\text{Beta}(\mu_1; \mu_2)$
• Time to failure (X) mean: $\mu_1 = 500$ hours
• Time to repair (Y): $\mu_2 = 30$ hours
• Generate $n = 5000$ random Beta values
  – with the above parameters $\mu_1$ and $\mu_2$
• Obtain the MC Availabilities: $A_i$
Histogram of Example

Frequency

Beta500-30

J. L. Romeu - Consultant (c) 2017
Estimated Parameters of Example

MC Results for Beta(500,30) Example:
Average Availability = 0.9435
Variance of Availability = 9.92x10^{-5}
Life $L_{10} = 0.9305$
Quartiles: 0.9370 and 0.9505
$P(A) > 0.9505 \approx 0.2694$

$$P\{A > 0.95\} = 1 - P\{A \leq 0.95\} \approx 1 - \frac{3673}{5000} = 1 - 0.7346 = 0.2654$$
Markov Model Approach

- Two-state Markov Chain (Refs. 4, 5, 6, 7)
- Monitor *status* of system at time T: X(T)
- Denote State 0 (Down), and State 1 (Up)
- Let X(T) = 0: system S is down at time T
- Define the probability “q” that system S is Up at time T (or “p”, that S was Down at T) given that it was Down (or Up) at time T-1?
Markov Representation of S:

\[ p_{01} = P\{ X(T) = 1 \mid X(T - 1) = 0 \} = q \]

\[ p_{10} = P\{ X(T) = 0 \mid X(T - 1) = 1 \} = p \]
Numerical Example:

• System S is in state Up; then moves to state Down in one step, with Prob. \( p_{10} = p = 0.002 \)
  – A Geometric distribution with Mean \( \mu = 1/p = 500 \) hours.

• System S is in state Down; then moves to state Up in one step, with Probability \( p_{01} = q = 0.033 \)
  – A Geometric distribution, with Mean \( \mu = 1/q = 30 \) hours.

• Every step (time period to transition) is an hour.

• The Geometric Distribution is the Discrete counterpart of the Continuous Exponential
Transition Probability Matrix $P$

<table>
<thead>
<tr>
<th>States</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$(1-q,q)$</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>$(p,1-p)$</td>
<td>1</td>
</tr>
</tbody>
</table>

Entries of Matrix $P = (p_{ij})$ correspond to the Markov Chain’s one-step transition probabilities. Rows represent every system state that $S$ can be in, at time $T$. Columns represent every other state that $S$ can go into, in one step (i.e. where $S$ will be, at time $T+1$).
Obtain the probability of S moving from state Up to Down, in Two Hours

\[ P^2 = \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix}^2 = \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix} \times \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix} \]

\[ = \begin{bmatrix} (1-q)^2 + pq & q(1-q) + q(1-p) \\ p(1-q) + p(1-p) & pq + (1-p)^2 \end{bmatrix} = \begin{bmatrix} p_{oo}^{(2)} & p_{01}^{(2)} \\ p_{10}^{(2)} & p_{11}^{(2)} \end{bmatrix} \]

\[ \Rightarrow p_{10}^{(2)} = p_{10} p_{00} + p_{11} p_{10} = p(1-q) + (1-p)p \]

Hence, the probability S will go down in two hours is:

\[ p_{10}^{(2)} = p(1-q) + (1-p)p = 0.003 \]
Other useful Markov results:

- If \( p_{10}^{(2)} = 0.003 \Rightarrow p_{11}^{(2)} = 1-p_{10}^{(2)} = A(T) = 0.993 \)
  - system Availability, after \( T=2 \) hours of operation

- Prob. of moving from state 1 to 0, in 10 steps:
  - \((P)^{10} \Rightarrow p_{10}^{(10)} = 0.017\); includes that S could have gone Down or Up, then restored again, several times.

- For sufficiently large \( n \) (long run) and two-states:

\[
\text{Limit}_{n \to \infty} P^n = \text{Limit}_{n \to \infty} \left\{ \frac{1}{p+q} \begin{bmatrix} p & q \\ p & q \end{bmatrix} + (1-p-q)^n \begin{bmatrix} q & -q \\ -p & p \end{bmatrix} \right\} = \begin{bmatrix} p/(p+q) & q/(p+q) \\ p/(p+q) & q/(p+q) \end{bmatrix}
\]

Example: Up = \( q/(p+q) = 0.943 \); Down = \( p/(p+q) = 0.057 \)
Markov Model for redundant system

• A Redundant System is composed of two identical devices, in parallel.
• The System is maintained and can function at a degraded level, with only one unit UP.
• The System has now three States: 0, 1, 2:
  – State 0, the **Down** state; both units are DOWN
  – State 1, the **Degraded** state; only one unit is UP
  – State 2, the **UP** state; both units are operating
Markov Model system representation

\[ p_{01} = P\{X(T) = 1 \mid X(T-1) = 0\} = q \]
\[ p_{10} = P\{X(T) = 0 \mid X(T-1) = 1\} = p \]
\[ p_{12} = P\{X(T) = 2 \mid X(T-1) = 1\} = q \]
\[ p_{21} = P\{X(T) = 1 \mid X(T-1) = 2\} = 2p \]
\[ p_{ii} = P\{X(T) = i \mid X(T-1) = i\} = 1 - \sum_{j \neq i} p_{ij} \]
Operational Conditions

• Every step (hour) $T$ is an independent trial
• Success Prob. $p_{ij}$ corresponds to a transition from current state ‘$i$’ into state ‘$j$’ = 0,1,2
• Distribution of every change of state is the Geometric (Counterpart of the Exponential)
• Mean time to accomplishing such change of state is: $\mu = 1/p_{ij}$
Transition Probability Matrix $P$:

$$
P = \begin{bmatrix}
0 & p_{00} & p_{01} & p_{02} \\
1 & p_{10} & p_{11} & p_{12} \\
2 & p_{20} & p_{21} & p_{22}
\end{bmatrix} = 
\begin{bmatrix}
0 & 1 - q & q & 0 \\
1 & p & 1 - p - q & q \\
2 & 0 & 2p & 1 - 2p
\end{bmatrix}
$$

As before, the probability of being in state “$j$” after “$n$” steps, given that we started in some state “$i$” of $S$, is obtained by raising matrix $P$ to the power “$n$”, and then looking at entry $p_{ij}$ of the resulting matrix $P^n$. 
Numerical Example

• Probability $p$ of either unit failing
  – in the next hour is 0.002

• Probability $q$ of the repair crew completing
  – a maintenance job in the next hour is 0.033

• Only one failure is allowed
  – in each unit time period,

• and only one repair can be undertaken
  – at any unit time
Probability that a degraded system (in State 1) remains degraded after two hours of operation:

- Sum probabilities corresponding to 3 events
  - the system status has never changed.
  - one unit repaired but another fails during 2\textsuperscript{nd} hour
  - remaining unit fails in the first hour (system goes down), but a repair is completed in the 2\textsuperscript{nd} hour
Numerical Example:

\[ P_{11}^2 = [P \times P]_{11} = p_{11}^{(2)} = p_{10} p_{01} + p_{11} p_{11} + p_{12} p_{21} \]
\[ = pq + (1 - p - q)^2 + 2pq \]
\[ = 0.002 \times 0.033 + (1 - 0.035)^2 + 2 \times 0.002 \times 0.033 \]
\[ P_{11}^2 = 0.9314 \]

The probability that a system, in degraded state, is still in degraded state after two hours, is: \( P_{11}^2 = 0.9314 \)
Mean time $\mu$ that the system S spends in the Degraded state

- System S can change to Up or Down
  - with probabilities $p$ and $q$, respectively
- S will remain in the state Degraded
  - with probability $1 - p - q$ (i.e. no change)
- On average, S will spend a “sojourn” of
  - length $1/ (p + q) = 1/ 0.035 = 28.57$ hrs
  - in the Degraded state, before moving out.
Availability at time T

- \( A(T) = P\{S \text{ is Available at } T\} \)
- System S is not Down at time “T”
  - Then, S can be either Up, or Degraded
- \( A(T) \) depends on the initial state of S
- Find Prob. S is “Degraded Available” at T
  - given that S was Degraded (initially 1) at \( T=0 \)

\[
p_{10}^{(T)} + p_{11}^{(T)} + p_{12}^{(T)} = 1 \Rightarrow p_{11}^{(T)} = 1 - p_{10}^{(T)} - p_{12}^{(T)}
\]

\[
A(T) = P\{X(T) = 1 \mid X(0) = 1\} = p_{11}^{(T)} = 1 - p_{10}^{(T)} - p_{12}^{(T)}
\]
State Occupancies

• Long run averages of system sojourns
• Asymptotic probabilities of system S being
  – in each one of its possible states at any time T
• Or the percent time S spent in these states
  – Irrespective of the state S was in, initially.
• Results are obtained by considering
  – Vector \( \Pi \) of the “long run” probabilities:
Characteristics of Vector П

\[ \Pi = \text{Limit}_{T \to \infty} \{ \text{Prob}\{X(T) = 0\}, \text{Prob}\{X(T) = 1\}, \text{Prob}\{X(T) = 2\}\} \]

Vector П fulfills two important properties:

1. : \( \Pi \times P = \Pi \);
2. \( \sum \Pi_i = 1 \); with : \( \Pi_i = \text{Limit}_{T \to \infty} \text{Prob}\{X(T) = i\} \)

\( \Pi \times P = \Pi \) (Vector П times matrix P equals П) defines a system of linear equations, “normalized” by the 2nd property.

\[
\Pi \times P = \begin{pmatrix} \Pi_0 & \Pi_1 & \Pi_2 \end{pmatrix} \times \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = \begin{pmatrix} \Pi_0 & \Pi_1 & \Pi_2 \end{pmatrix}
\]

with: \( \sum_i \Pi_i = \Pi_0 + \Pi_1 + \Pi_2 = 1 \)
Numerical Example

\[ \Pi \times P = (\Pi_0, \Pi_1, \Pi_2) \times \begin{bmatrix} 0.967 & 0.033 & 0 \\ 0.002 & 0.965 & 0.033 \\ 0 & 0.004 & 0.996 \end{bmatrix} = (\Pi_0, \Pi_1, \Pi_2) \]

\[ \begin{align*}
0.967\Pi_0 + 0.002\Pi_1 &= \Pi_0 \\
0.033\Pi_0 + 0.965\Pi_1 + 0.004\Pi_2 &= \Pi_1; \text{ with } \sum_i \Pi_i = \Pi_0 + \Pi_1 + \Pi_2 = 1 \\
0.033\Pi_1 + 0.996\Pi_2 &= \Pi_2
\end{align*} \]

Solution of the system yields long run occupancy rates:

\[ \Pi = (\Pi_0, \Pi_1, \Pi_2) = (0.0065, 0.1074, 0.8861) \]
Interpretation of results:

- $\Pi_2 = 0.8861$ indicates that system $S$
  - is operating at full capacity 88% of the time.
- $\Pi_1 = 0.1074$ indicates that system $S$
  - is operating at Degraded capacity 10% of the time.
- $\Pi_0$: probability corresponding to State 0 (Down)
  - is associated with $S$ being Unavailable ($= 0.0065$)
- “long run” System Availability is given by:
  - $A = 1 - \Pi_0 = 1 - 0.0065 = 0.9935$
Expected Times

- For System S to go Down, if initially
  - S was Up (denoted $V_2$), or Degraded ($V_1$)
- Or the average time System S spent in each
  - of these states (1, 2) before going “Down”.
- Assume Down is an “absorbing” state
  - one that, once entered, can never be left
- Solve a system of equations leading to
  - all such possible situations.
Numerical Example:

One step, at minimum (initial visit), before system S goes Down. If S is not absorbed then, system S will move on to any of other, non-absorbing (Up, Degraded) state with corresponding probability, and then the process restarts:

\[ V_1 = 1 + p_{11}V_1 + p_{12}V_2 = 1 + 0.965V_1 + 0.033V_2 \]
\[ V_2 = 1 + p_{21}V_1 + p_{22}V_2 = 1 + 0.004V_1 + 0.996V_2 \]

Average times until system S goes down yield:
\[ V_1 = 4625 \] (if starting in state Degraded) and \[ V_2 = 4875 \] (if starting in state Up).
Model Comparisons

• The initially non-maintained system version,
  – would work an Expected \( 3/2\lambda = 3/0.004 = 750 \)
  – hours in Up state, before going Down (Ref. 7).

• The fact that maintenance is now possible, and
  that S can operate in a Degraded state:
  – results in an increase of \( \mu/2\lambda^2 = 0.033/2 \times 0.002^2 = 4125 \)
  – hours in its Expected Time to go Down (from Up).

• The improved Expected Time is due to the
  Sum of the Two Expected times to failures:
  – \( V_2 = 3/2\lambda + \mu/2\lambda^2 = 750 + 4125 = 4875 \)
Example of Increasing Availability

\[
A = \frac{MTBF}{MTBF + MTTR} = \frac{85}{85 + 15} = 0.85 \approx 85\%
\]

- Assume MTTR is largely affected by delays:
  - Waiting for a specialist mechanic
  - Waiting for a special spare part
- Assume Availability HAS to be at least 90%:
  - We can hire additional specialists or mechanics
  - We can increase the warehouse parts inventory
- Assume such would reduce MTTR to 8 units:
  - Therefore, the New System Availability is:
    \[
    A = \frac{85}{85 + 8} = \frac{85}{93} = 0.914 \approx 91.4\%
    \]
Conclusions

• Availability is the ratio of:
  – $Up.Time$ to $Cycle.Time$

• Hence, we can enhance Availability by:
  – Increasing the device or system Life (R)
  – Decreasing/Improving maintenance time
  – Simultaneously, doing both above.

• Decreasing maintenance is usually:
  – Easier and/or Cheaper.
Juarez Lincoln Marti Int’l Ed. Project

- The JLM International Project develops programs to support Higher Education in Iberoamerica
- Its Web Page: http://web.cortland.edu/materesearch/
- A quick overview of the Project in PPT:
- JLM Project Sponsors the Quality, Reliability and Industrial Statistics Institute Web Site:
  - http://ecs.syr.edu/faculty/romeu/QR&CII.htm

This paper is available in the Internet: http://src.alionscience.com/pdf/AVAILSTAT.pdf
Bibliography


About the Author

**Jorge Luis Romeu** is a Research Professor, Syracuse University (SU), where he teaches statistics, quality and operations research courses. He worked as a Senior Engineer for the RIAC (Reliability Information Analysis Center). Romeu has 40 years applying statistical and operations research methods to HW/SW reliability, quality and industrial engineering. Romeu retired Emeritus from SUNY, where he taught mathematics, statistics and computers. He was a Fulbright Senior Specialist, at universities in Mexico (1994, 2000 and 2003), Dominican Republic (2004), and Ecuador (2006). He created and directs the Juarez-Lincoln-Martí Int’l Ed. Project. Romeu is lead author of *A Practical Guide to Statistical Analysis of Materials Property Data*. He has developed and teaches many workshops and training courses for practicing engineers and statistics faculty, and has published over forty articles on applied statistics and statistical education. He obtained the Saaty Award for the Best Applied Statistics Paper in *American Journal of Mathematics and Management Sciences* (AJMMS), in 1997 and 2007, and the MVEEC Award for Outstanding Professional Development in 2002 & 2012. Romeu holds a Ph.D. in Operations Research, is a Chartered Statistician Fellow of the Royal Statistical Society, member of the American Statistical Association, and a Senior Member of the American Society for Quality. He holds ASQ certifications in Quality and Reliability, He is Past Regional Director of ASQ Region II and Vice-Chair of NEQC. For more information, visit his web site in http://web.cortland.edu/romeu