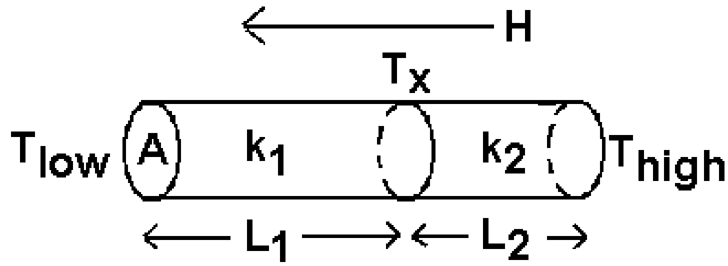
Handout 1 - Heat Transfer in a Composite Rod - Day 3



The heat transfer must be the same throughout the rod at equilibrium, therefore heat transfer through the first rod must equal heat transfer through the second rod, i.e.

$$H = \frac{k_2 A}{L_2}\left(T_H - T_x\right) = \frac{k_1 A}{L_1}\left(T_x - T_L\right) \quad \Rightarrow \quad L_1 k_2\left(T_H - T_x\right) = L_2 k_1\left(T_x - T_L\right)$$

or $\quad T_x = \dfrac{L_1 k_2 T_H + L_2 k_1 T_L}{L_1 k_2 + L_2 k_1}$. We plug this back into the first equation to obtain

$$H = \frac{A\left(T_H - T_L\right)}{\frac{L_1}{k_1} + \frac{L_2}{k_2}} = \frac{A\left(T_H - T_L\right)}{R_1 + R_2}$$, where R is the thermal resistance of each component.

In general, $H = \dfrac{A\left(T_H - T_L\right)}{\sum R_i}$, which looks the same for series resistors.

Example: $L_1 = L_2 = .1$ m, equal lengths; $T_{high} = 100$ °C and $T_{low} = 0$ °C;
k_1 = iron = 79.5 W/m°C, k_2 = lead = 34.7 W/m°C.

What is T_x, the midway temperature? Note: this problem need not be in Kelvin.

$$T_x = \frac{L_1 k_2 T_H + L_2 k_1 T_L}{L_1 k_2 + L_2 k_1} = \frac{k_2 T_H + k_1 T_L}{k_2 + k_1} = \frac{k_2}{k_2 + k_1} T_H$$

$$= \frac{34.7}{34.7 + 79.5} 100\ {}^{o}\mathrm{C} = 30.4\ {}^{o}\mathrm{C}$$

This is what we expect. Lead is the poorer conductor with lower thermal conductivity and higher thermal resistivity; thus, for the same length and same heat transfer it must have a larger thermal gradient.