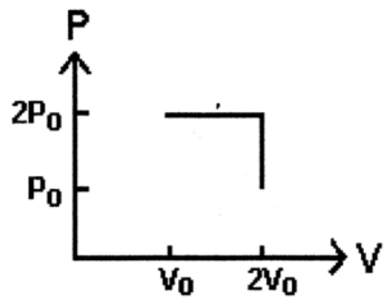
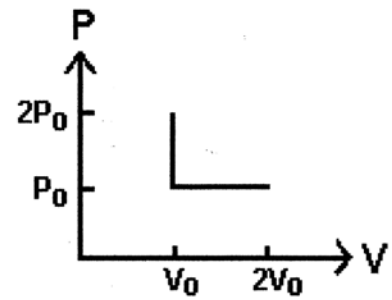
Problem: What is the difference in work done and heat added to one mole of an ideal monatomic gas taken from state (2Po,Vo, To) to state (Po, 2Vo, T_f) by the two processes shown.





1) In the first process the gas expands isobarically, then is reduced to half the pressure isovolumetrically.

<u>In the isobaric process:</u> $W = P\Delta V = 2P_o(2V_o - V_o) = 2P_oV_o$

At the same time, since the pressure and number of moles of gas are held constant

$$\Delta(PV) = \Delta(nrT) \quad \Rightarrow \quad \frac{\Delta V}{V} = \frac{\Delta T}{T} \quad \Rightarrow \quad T_{intermediate} = 2T_o$$

$$Q = nC_P\Delta T = C_P(2T_o - T_o) = \frac{5}{2}RT_0 = \frac{5}{2}(2P_0)V_0 = 5P_0V_0$$

$$\Delta U = Q - W = 3P_oV_o$$

<u>In the isovolumetric process:</u> $W = 0$

At the same time, since the pressure and number of moles of gas are held constant

$$\Delta(PV) = \Delta(nrT) \quad \Rightarrow \quad \frac{\Delta P}{P} = \frac{\Delta T}{T} \quad \Rightarrow \quad T_{final} = T_o$$

This seems obvious since $\Delta(PV) = 0$ for the two-step process.

$\Rightarrow$ ΔT in the two-step process is zero.

$$\Delta U = Q = nC_V\Delta T = C_V(T_o - 2T_o) = -\frac{3}{2}RT_0 = -\frac{3}{2}(2P_0)V_0 = -3P_0V_0$$

The net result is
net work done by the gas = $2P_oV_o$
net heat added to the gas = $2P_oV_o$
Net change in energy = 0

2) In the second process the gas is reduced to half the initial pressure isovolumetrically, then expands isobarically.

<u>In the isovolumetric process:</u> $W = 0$

At the same time, since the pressure and number of moles of gas are held constant

$$\Delta(PV) = \Delta(nrT) \Rightarrow \frac{\Delta P}{P} = \frac{\Delta T}{T} \Rightarrow T_{intermediatel} = \frac{T_0}{2}$$

$$\Delta U = Q = nC_V\Delta T = C_V(\frac{T_0}{2} - T_o) = -\frac{3}{2}R\frac{T_0}{2} = -\frac{3}{4}(2P_0)V_0 = -\frac{3}{2}P_0V_0$$

<u>In the isobaric process:</u> $W = P\Delta V = P_o(2V_o - V_o) = P_oV_o$

At the same time, since the pressure and number of moles of gas are held constant

$$\Delta(PV) = \Delta(nrT) \Rightarrow \frac{\Delta V}{V} = \frac{\Delta T}{T} \Rightarrow T_{final} = T_o$$

This seems obvious since $\Delta(PV) = 0$ for the two-step process.

$\Rightarrow$ ΔT in the two-step process is zero.

$$Q = nC_P\Delta T = C_P(T_o - \frac{T_0}{2}) = \frac{5}{2}R\frac{T_0}{2} = \frac{5}{4}(2P_0)V_0 = \frac{5}{2}P_0V_0$$

$$\Delta U = Q - W = \frac{3}{2}P_oV_o$$

The net result is
net work done by the gas = P_oV_o
net heat added to the gas = P_oV_o
Net change in energy = 0

We conclude that more work is done in the first process and more heat added than in the second process.